

# The Effect of Pay-It-Forward during Disasters on Social Networks: A Network Approach

Riku Tanimoto\*, Hitomu Kotani\*\*

\*Department of Urban Management, Graduate School of Engineering, Kyoto University,  
C1, Nishikyo-ku, Kyoto, 615-8540, Japan  
E-mail: tanimoto.riku.66m@st.kyoto-u.ac.jp

\*\* Department of Civil and Environmental Engineering, School of Environment and Society, Tokyo Institute of Technology  
Ookayama, Meguro-ku, Tokyo, 152-8550, Japan  
E-mail: kotani.h.ac@m.titech.ac.jp

Some disaster survivors who had previously received support through volunteering participated in volunteer activities for people affected by subsequent disasters, which was observed after the 2011 Great East Japan Earthquake and the 2016 Kumamoto Earthquake in Japan. This chain of support, known as “pay-it-forward,” is expected to expand social networks and bring about small-world property (i.e., networks with high clustering and short path length between people), as suggested by Atsumi [1]. Most studies have focused on the effects of pay-it-forward on individuals (i.e., psychological perspective), but studies have insufficiently determined its effects on the whole society or social networks (i.e., sociological perspective). This study aims to investigate the dynamic effects of pay-it-forward on network properties. We proposed a network formation model considering pay-it-forward during disasters and conducted numerical simulations. The results showed that pay-it-forward led to small-world property and higher social welfare in the long term, because it eliminated the disparity in ties between people (in particular, it reduced the number of people with fewer ties), which accelerated network formation during the period without disasters. This result was more pronounced in societies with a larger disparity in ties. From a sociological perspective, our findings imply the significance of pay-it-forward volunteering and volunteer organizations that promote such activities.

**Keywords:** small world, volunteers, upstream reciprocity, network formation

## 1. Introduction

The various volunteer activities are implemented in times of disaster, among which pay-it-forward (PF) has been attracting attention from psychological and sociological perspectives. Pay-it-forward refers to the chain of actions where person A supports person B in a disaster area, and B supports person C in a

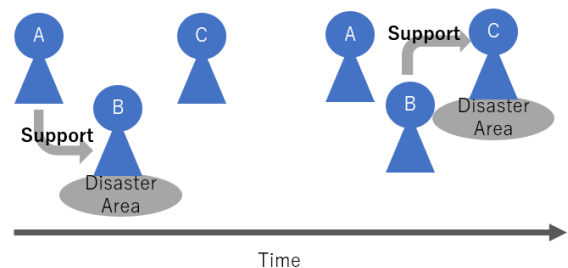


Fig. 1. Example of pay-it-forward

subsequent disaster area (Fig. 1) [1]. This kind of return of favor to another person (person C for B), rather than directly to the one who helped (person A for B), is also known as “upstream reciprocity” in “indirect reciprocity” [2]. Pay-it-forward was observed in past disasters such as the 2011 Great East Japan Earthquake and the 2016 Kumamoto Earthquake in Japan [1,3].

Two types of effects of pay-it-forward have been pointed out previously [1]: 1. the psychological effects on past disaster relief recipients (person B in Fig. 1) and 2. the sociological effects on the entire society. Much of the research is on the former, which mainly studied the feeling of indebtedness of past disaster survivors [1]. Many studies have shown through interviews that people feel a sense of indebtedness when they receive disaster relief [1,4]. Some empirical studies tested that the debt from receiving assistance leads to subsequent disaster volunteering [5,6] and other altruistic behaviors [7]; others tested that pay-it-forward reduces the indebtedness of relief providers [1,4].

In terms of sociological effects, Atsumi [1] suggests that an increase in near-random ties, such as pay-it-forward, may bring small-world property to social networks, as shown by Watts and Strogatz [8]. However, research on the sociological effects of pay-it-forward has mainly focused on its effects under static social networks. The examples include theoretical studies examining the effects of upstream reciprocity on the evolution of cooperative behavior using evolutionary games on static networks [9,10,11] and statistical studies investigating the

interdependence between pay-it-forward and cooperative behavior in a static network [12]. However, to the best of our knowledge, there are no studies on the effect of pay-it-forward on the network properties in dynamic network formation processes.

This paper aims to examine the dynamic effects of pay-it-forward on network properties. To this end, we formulate a social network model that considers pay-it-forward and conduct a numerical simulation. We capture the pay-it-forward effects in terms of small-world property. Watts and Strogatz [8] described the small-world property as an “idea that we are all connected to each other via very short paths, typically encompassing only a handful of intermediaries,” and illustrated it with cluster coefficients and average distance lengths. It has been empirically shown that small-world property realizes a society in which information and knowledge transfer is faster, trust is easily formed, innovation is more likely to emerge, and productivity is higher [13,14,15]. The small-world property likely brings significant social benefits. This study examines whether pay-it-forward brings small-world property in networks (in terms of cluster coefficient and distance length) and, if so, in what kind of mechanism it brings.

This study contributes to evaluating pay-it-forward from the sociological perspective by showing the long-term effects of pay-it-forward on social networks. This evaluation implies the significance of the work of volunteer organizations (e.g., non-profit organizations (NPOs)) that promote pay-it-forward volunteer activities (i.e., coordinate the participation of affected people of past disasters in volunteer efforts for other disasters) [1] and the rationale for promoting their work.

## 2. Method: Network Formation Model Considering Pay-It-Forward

### 2.1. Utility Function

Relationships among individuals (hereafter “players”) are represented by graphs. Let  $N = \{1, \dots, n\}, n > 2$  be the players of the network. A node represents a player, and a link represents an interaction between players. Let the network  $g$  be the set of links between players in  $N$ . The links between players  $i$  and  $j$  are denoted by  $ij$ , where  $ij \in g$  indicates that players  $i$  and  $j$  interact directly in network  $g$  and  $ij \notin g$  indicates that there is no interaction. Also,  $g + ij = g \cup \{ij\}$  indicates that link  $ij$  has been added to network  $g$ . Conversely,  $g - ij = g \setminus \{ij\}$  indicates that link  $ij$  has been removed from network  $g$ .

Neighbors of player  $i$  can be denoted by all  $j$  such that  $ij \in g$ . We assume that player  $i$  interacts with his neighbors and has a certain level of social interaction, such as exchanges of goods and information, transactions, and so on. The set of

neighbors of player  $i$  can be denoted by  $N_i(g) = \{j \in N | ij \in g\}$ . The degree of player  $i$  is the number of neighbors of  $i$ , which is defined as  $d_i(g) = |N_i(g)|$ .

The utility  $u_i(g)$  of player  $i$  in network  $g$  is defined as follows, following Kotani and Yokomatsu [16].

$$u_i(g) = \sum_{j \in N_i(g)} b(d_j) - c(d_i) \cdot d_i, \dots\dots\dots(1)$$

where

$$b(d_j) = d_j - 1, \dots\dots\dots(2)$$

$$c(d_i) = \bar{c} \cdot d_i, \dots\dots\dots(3)$$

The first term,  $b(d_j)$ , indicates the benefit obtained from neighbor  $j$ , and is a monotonically increasing function with respect to  $d_j$ . This indicates that the larger the degree of neighbor  $j$  is, the greater the amount of resources and information obtained through interaction with  $j$ , and the greater the utility. On the other hand, the second term  $c(d_i)$ , where  $\bar{c}$  is a constant, represents the interaction cost per player in the neighbors, which is a monotonically increasing function with respect to the degree of player  $i$ ,  $d_i$ . This indicates that as the degree increases, it becomes more difficult for  $i$  to adjust his/her schedule and the interaction cost increases.

### 2.2. Network Formation Process in Peaceful Times

The network formation process is based on the dynamic model proposed by Jackson and Watts [17], which has been used in many papers. Each player is assumed to have the following bounded rationality.

1. Not all players react immediately to their environment (the inertia hypothesis)
2. Each player reacts myopically when they react (the myopic hypothesis)
3. Each player randomly changes his/her strategy with a small probability (the error/mutation hypothesis)

The third, error/mutation, is an external factor independent of each player and represents a change in interaction relationships due to unexpected encounters, unemployment, illness, taking up a new hobby, and so on. The error/mutation causes the network to continue to change. Thus, the process allows us to identify the most robust or easy-to-reach networks in the long run.

The dynamic process in period  $t$  consists of the following four steps (steps 1 to 4). Let  $g^t$  denote the network at the beginning of step 1 in period  $t$  ( $t = \{1, \dots, T\}, T > 2$ ).

Step 1. A pair of players is randomly selected along with a constant probability distribution  $\{\pi_{ij}\}$  ( $\pi_{ij} > 0$  for all  $ij$ ).

Step 2. The remaining players (i.e., all players other than players  $i$  and  $j$  selected in step 1) do not react to the environment. That is, their links do not change (the inertia hypothesis).

Step 3. Under the myopic hypothesis, players  $i$  and  $j$  selected in step 1 decide whether to form or delete link  $ij$ , following steps 3-1 and 3-2 below. Steps 3-1 and 3-2 are called “two-sided link formation,” which means that forming a link requires the agreement of both players while severing a link can be done unilaterally [17]. Players  $i$  and  $j$  make their decision based on the assumption that all links except link  $ij$  is in the same state as at the beginning of the period.

Step 3-1. In the case of  $ij \notin g^t$

A link  $ij$  is formed if the utility of at least one of the players strictly increases and the utility of the other player does not decrease. Otherwise, they do not form it.

Step 3-2. In the case of  $ij \in g^t$

A link  $ij$  is disconnected if deleting link  $ij$  will strictly increase the utility of at least one of the players. Otherwise, they do not disconnect.

Step 4. Error/mutation occurs with a small probability  $\varepsilon$  ( $0 < \varepsilon < 1$ ). That is, with probability  $\varepsilon$ , the decision of step 3 is reversed; with probability  $1 - \varepsilon$ , the decision remains the same. This process generates  $g_1^t$ .

The above four steps can be summarized as follows.

<In the case of  $ij \notin g^t$  >

If  $u_i(g^t + ij) \geq u_i(g^t)$  and  $u_j(g^t + ij) \geq u_j(g^t)$  with one inequality strict, then  $g_1^t = g^t + ij$  with probability  $1 - \varepsilon$  and  $g_1^t = g^t$  with probability  $\varepsilon$ . Otherwise,  $g_1^t = g^t$  with probability  $1 - \varepsilon$  and  $g_1^t = g^t + ij$  with probability  $\varepsilon$ .

<In the case of  $ij \in g^t$  >

If  $u_i(g^t - ij) > u_i(g^t)$  and/or  $u_j(g^t - ij) > u_j(g^t)$ , then  $g_1^t = g^t - ij$  with probability  $1 - \varepsilon$  and  $g_1^t = g^t$  with probability  $\varepsilon$ . Otherwise,  $g_1^t = g^t$  with probability  $1 - \varepsilon$  and  $g_1^t = g^t - ij$  with probability  $\varepsilon$ .

Through this process, the network  $g_1^t$  at the end of period  $t$  is determined. This network becomes the network  $g^{t+1}$  at the beginning of period  $t + 1$ , and steps 1 to 4 are repeated in period  $t + 1$ .

### 2.3. Network Formation Process Considering Pay-It-Forward at the Time of Disaster

To model pay-it-forward, we assume that a disaster occurs with a constant probability  $\eta$  ( $0 < \eta < 1$ ) in the network formation process every period. If a disaster occurs, support activities such as volunteering may occur and new links may be formed. Let  $k$  be the number of times a new link is formed by pay-it-forward by the beginning of period  $t$  ( $k = 0, 1, 2, \dots$ ). Let  $p_k$  be the support provider (hereafter, simply

“provider”) in the pay-it-forward at event number  $k$  and  $r_k$  be the recipient of support, and assume that the recipient may become a provider in the subsequent disasters and help an affected player. In other words,  $r_k = p_{k+1}$ . Under these assumptions, the network formation process during a disaster is taken in the fifth step after the fourth step in peaceful times.

Step 5. Suppose that a disaster occurs with a certain probability  $\eta$ . If a disaster occurs, the following steps 5-1 and 5-2 determine whether or not pay-it-forward occurs. If no disaster occurs (with probability  $1 - \eta$ ), step 5 ends.

Step 5-1a. In the case of  $k = 0$

Of the pair  $ij$  selected in step 1, one of the players is affected by the disaster with equal probability for each player and is denoted by  $r'$  as the player who may receive support. The other player of pair  $ij$  is not affected by the disaster, who is denoted by  $p'$  as the player may support player  $r'$ .

Step 5-1b. In the case of  $k \geq 1$

Let  $r_k$  (i.e., the player who was the recipient in the most recent support) be the player  $p'$  who is likely to support in the current period ( $p' = r_k$ ). Then, let  $r'$  be the player who may be supported by one of the pair  $ij$  selected in step 1, with equal probability that one of the players is affected by a disaster. Note that if  $p'(= r_k) = r'$ , then we redefine the other player of  $ij$  as  $r'$ .

Step 5-2. If the degree of the player  $p'$  who may support is zero, there is no change in the network. If the degree of  $p'$  is not zero, and if there is no link between  $p'$  and  $r'$ , then with a probability  $1 - \varepsilon$ , player  $p'$  will provide support for player  $r'$ , from which a communication occurs and a new link is formed. When a new link is formed,  $p_{k+1} = p'$  and  $r_{k+1} = r'$ . Then, player  $p'$  selects one of his/her neighbors  $o$  with equal probability among the neighbors and delete link  $p'o$ .

The above process is summarized as follows.

$g_2^t = g_1^t$  happens with probability  $1 - \eta$ . Otherwise, the following process happens with probability  $\eta$ .

<In the case of  $d_{p'} \neq 0$ >

• In the case of  $p'r' \notin g_1^t$

$g_2^t = g_1^t + p'r' - p'o$ ,  $p' = p_{k+1}, r' = r_{k+1}$  with probability  $1 - \varepsilon$  and  $g_2^t = g_1^t$  with probability  $\varepsilon$ .

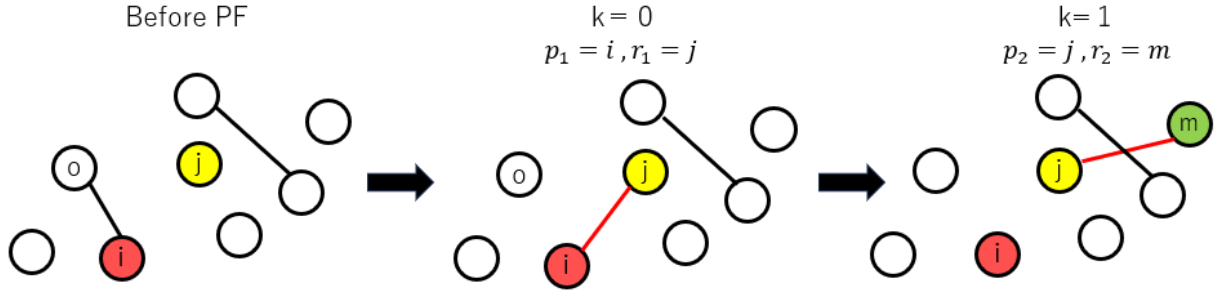
• In the case of  $p'r' \in g_1^t$

$g_2^t = g_1^t$

<In the case of  $d_{p'} = 0$  >

$g_2^t = g_1^t$

Through this process, the network  $g_2^t$  at the end of period  $t$  is determined. This network becomes the network  $g^{t+1}$  at the beginning of period  $t + 1$ . In period  $t + 1$ , if a disaster occurs, steps 1 to 5 are repeated.



**Fig. 2.** Schematic diagram of pay-it-forward in our algorithm

An example of pay-it-forward in this algorithm is shown below (**Fig. 2**). Suppose that when  $k = 0$  (i.e., no pay-it-forward has ever occurred in the past), player  $i$  as a provider form a link  $ij$  with player  $j$  as a recipient ( $p_1 = i$  and  $r_1 = j$ ) (link  $ij$  in the network in the middle of **Fig. 2**). In this case, the provider  $i$  is assumed to randomly disconnect one of the already existing links (link  $io$  in the network in the middle of **Fig. 2**). Then, when  $k = 1$ , player  $j$ , who was a recipient in the pay-it-forward when  $k = 0$ , becomes a provider ( $p_2 = r_1 = j$ ) and connects a new link (link  $jm$  in the network on the right-hand side of **Fig. 2**). Note that **Fig. 2** omits the network formation process of peaceful time between  $k = 0$  and  $k = 1$ ; however, in the algorithm, as the network formation process of peaceful time exists, player  $j$  may delete a link with other than player  $i$  if player  $j$  has more than one link when  $k = 1$ .

There are two reasons why we assume that provider  $p_k$  deletes a link at random among the neighbors in step 5-2. The first reason is that we assume that when helping an affected player, the provider has to reduce the physical and temporal resources to interact with existing neighbors, which leads him to delete an existing link. The second reason is to make a clear comparative analysis of the effects of pay-it-forward. If the existing links are not deleted, the degree of the provider  $p_k$  will necessarily increase after the support provision, and this will obviously increase the total number of links in the network. In contrast, our algorithm keeps the degree of the provider  $p_k$  same before and after the support. It prevents us from obtaining the obvious result and allows us to investigate the true long-term effect of pay-it-forward.

#### 2.4. Parameter Settings

We set the parameters as follows, largely following Kotani and Yokomatsu [16]. The number of players is  $n = 20$ , and the network formation process is performed for  $T = 3000$  periods. The selection probability of players in step 1 is assumed to be

uniformly distributed ( $\pi_{ij} = \frac{1}{\binom{n}{2}}$ ). The interaction cost in the utility function is  $\bar{c} = 0.2$ . The probability of error is  $\varepsilon = 0.05$ , and the probability of disaster occurrence is  $\eta = 0.1$ . The initial network is an empty network where all players have no links ( $g^1 = \emptyset$ ). We run a Monte Carlo simulation with 1000 iterations and analyzed the resulting Monte Carlo average.

#### 2.5. Evaluation Indicators

We use the average clustering coefficient (ACC) and average of inverse distance (AID) as indicators of the small-world property. ACC is defined as the average of the clustering coefficient:  $Cl_i(g)$ , which indicates the proportion of links formed among player's neighbors [8]. According to Coleman [18], the cluster is important because, for example, it reduces betrayal due to common acquaintances.

$$ACC = \frac{\sum_{i \in N} Cl_i(g)}{n}, \dots\dots\dots(4)$$

where

$$Cl_i(g) = \frac{\#\{jk \in g | j \neq k, j \in N_i(g), k \in N_i(g)\}}{\#\{jk | j \neq k, j \in N_i(g), k \in N_i(g)\}} \dots\dots\dots(5)$$

AID measures the path length of a network, and it is especially suitable when there is more than one component<sup>1</sup>. It can be defined as follows [19,20].

$$AID = \frac{\sum_{i \in N} L_i(g)}{n}, \dots\dots\dots(6)$$

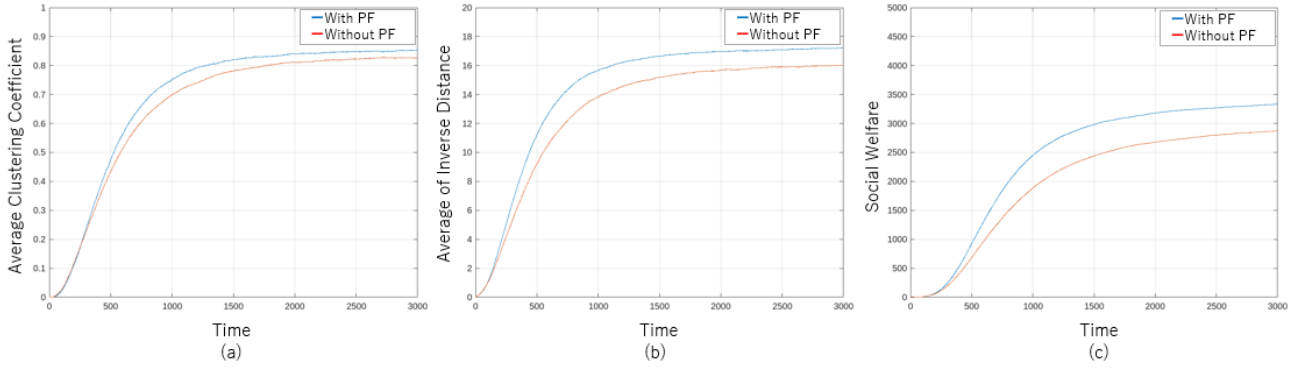
where

$$L_i(g) = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{l_{ij}} \dots\dots\dots(7)$$

$l_{ij}$  is the path length from player  $i$  to  $j$ , i.e., the number of links in the shortest path.  $l_{ij} = \infty$  if there is no path between  $i$  and  $j$  (i.e., they are not in the same component). The larger  $L_i(g)$  is, the smaller the path length player  $i$  has to the other players. AID is the average of  $L_i(g)$  (i.e., the inverse of distance)

<sup>1</sup> Components are maximal subnetworks such that every pair of nodes in the subnetwork is connected by a sequence of links (M. O. Jackson, "Social and

Economic Networks". Princeton University Press, 2008)



**Fig. 3.** Comparative dynamics of (a)ACC, (b)AID, and (c)SW

and a higher AID indicates a shorter path length between players in the whole social network. The higher ACC and AID are, the higher the clustering and the closer the distance between players are, respectively, which means that the network is more likely to be a small world.

We also define social welfare (SW) as follows, and we analyze its change due to pay-it-forward.

$$SW = \sum_{i \in N} u_i(g) \dots\dots\dots(8)$$

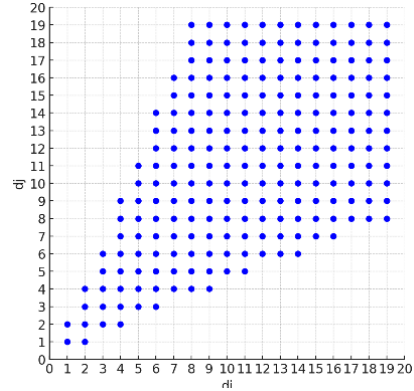
We simulated network formation with and without pay-it-forward and performed comparative dynamics of the Monte Carlo average in ACC, AID, and SW.

### 3. Results

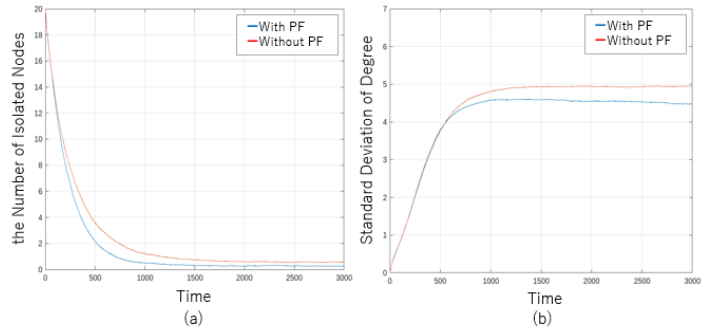
#### 3.1. Small-World Property Due to Pay-It-Forward

**Fig. 3(a)**, **Fig. 3(b)**, and **Fig. 3(c)** represent the transition of ACC, AID, and SW, respectively, where the red line shows the case without pay-it-forward, while the blue line shows that with pay-it-forward. **Fig. 3(a)** and **Fig. 3(b)** show that ACC and AID are larger with pay-it-forward from around  $t = 250$  than those without. This result clearly indicates that pay-it-forward brings the small-world property. **Fig. 3(c)** shows that the SW is also larger with pay-it-forward than without pay-it-forward, which indicates that pay-it-forward leads to higher SW.

Pay-it-forward brought about the small-world property probably because it eliminated the disparity in degree, which we call the “degree disparity elimination effect” in the following. **Fig. 4** shows the combinations of degrees in which links are formed in two-sided link formation in step 3 under peaceful times. This figure shows that there are only a limited number of combinations that can form a link, and in step 3, players with degree of one or higher do not form a link with an isolated player, and such pairs can only have a link through error/mutation in step 4. On the other hand, pay-it-forward enables such a pair to have a link even if the recipient’s degree is zero (isolated player), so the number of isolated players decreases quickly. In fact, **Fig. 5(a)**, which shows the transition of the number of isolated players, represents



**Fig. 4.** Combinations of degrees of players in which link formation occurs in peaceful times



**Fig. 5.** Comparative dynamics of (a) the number of isolated players and (b) standard deviation of degree

that the decrease in the number of isolated players with pay-it-forward is accelerated at around  $t = 250$  compared to the case without pay-it-forward. The decrease in isolated players likely accelerates the link formation during the subsequent peaceful time, resulting in small-world property.

**Fig. 4** also shows that in step 3 of peaceful time, players do not form a link when they have a large degree difference. Instead, only players with close degrees form a link, further increasing the degree disparity. On the other hand, during a disaster described in Section 2.3, links can be formed regardless of each other’s degree, facilitating the elimination of degree disparity and promoting link formation during peaceful times. It results in larger ACC and AID. In fact, **Fig. 5(b)**, which shows the transition of the standard deviation of degree, shows that the standard deviation increases until around  $t =$

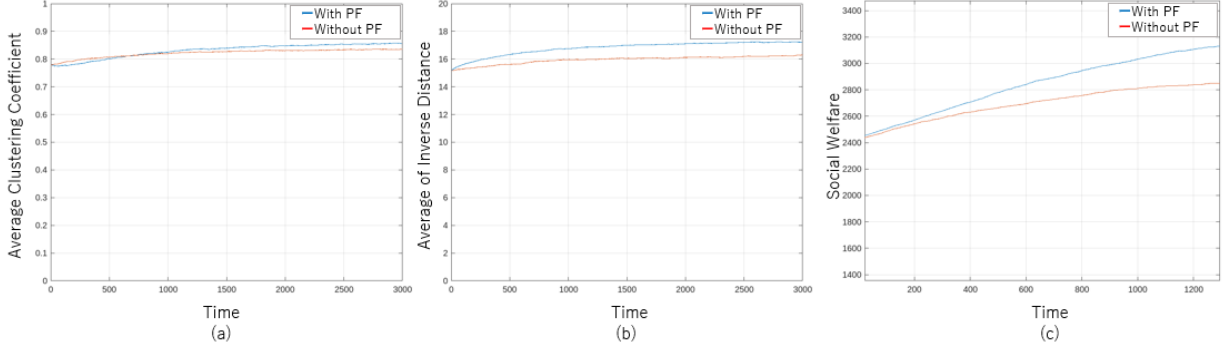


Fig. 6. Comparative dynamics of (a)ACC, (b)AID, and (c)SW when the initial network is  $\hat{g}$

500 both with and without pay-it-forward, but after that, the standard deviation of degree becomes smaller with pay-it-forward than that without pay-it-forward.

In summary, pay-it-forward decreases the number of isolated players dramatically between around  $t = 250$  and  $500$ , Once the number of isolated players decreases and the degree disparity starts to be large (i.e., from approximately  $t = 500$ ), the standard deviation of degree decreases due to pay-it-forward. Accordingly, it is considered that the “degree disparity elimination effect” accelerates network formation during peaceful times, resulting in larger ACC, AID, and SW.

### 3.2. Effects of Pay-It-Forward When Changing the Initial Network

In order to analyze the mechanism of the long-term effect of pay-it-forward in more detail, we vary the initial network  $g^1$ . As the initial network, we use the network  $\hat{g} = g^{1500}$ , which was formed in  $t = 1500$  only during peaceful times. Fig. 7 shows the histogram of degrees for 1000 networks,  $\hat{g} = g^{1500}$ , generated by Monte Carlo simulation.

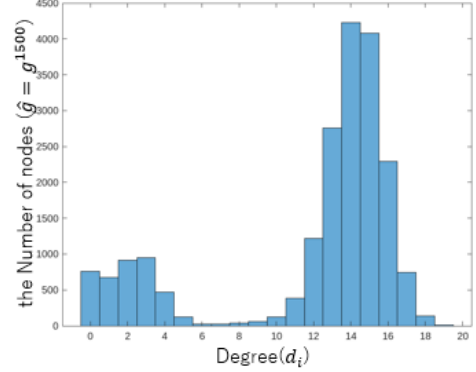


Fig. 7. the histogram of degree distribution when the initial network is  $\hat{g}$

Fig. 6(a) shows the transition of the Monte Carlo average of ACC when the initial network is  $\hat{g}$ . The blue and red lines mean the cases with and without pay-it-forward, respectively. According to the figure, ACC is lower with pay-it-forward than without pay-it-forward, around for  $t < 500$ . This is probably because pay-it-forward allows pairs with a significant degree disparity, in which they have difficulty forming a link during peaceful times, to form links through rewiring existing links. This rewiring possibly eliminates clusters.

In contrast, after around  $t = 800$ , ACC is higher in the case with pay-it-forward than in that without pay-it-forward. This is probably due to the fact that player with lower degree increases their degree over time, and the gap of degree with the higher-degree player decreases (i.e., degree disparity elimination effect). Therefore, link formation during peaceful times is promoted, and cluster formation is accelerated.

Fig. 8(a) shows the Monte Carlo average of the degree transition with and without pay-it-forward when we classify players with  $d_i < 8$  as Group 1 and those with  $d_i \geq 8$  as Group 2 at time  $t = 1$ . This

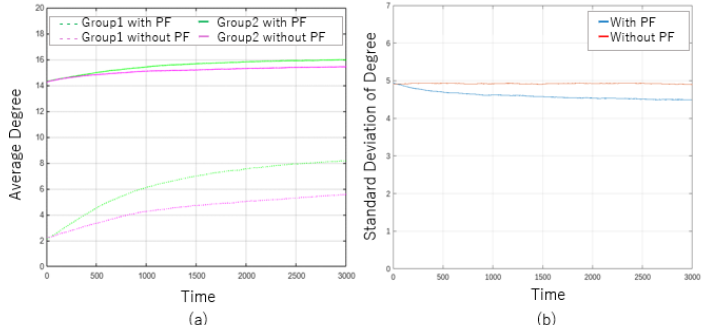


Fig. 8. Comparative dynamics of (a) average degree of Group 1 ( $d_i < 8$ ) and Group 2 ( $d_i \geq 8$ ) and (b) standard deviation of degree when the initial network is  $\hat{g}$

figure shows that the average degree of players in Group 1 increases faster than that in Group 2, which reduces the standard deviation of the degree, as shown in Fig. 8(b). According to Fig. 4, players whose degree exceeds eight are more likely to form links with players whose degree is up to 19. These facts indicate that although the pay-it-forward temporarily suppressed the increase in ACC, ACC increased in the long term as the degree disparity decreased and link formation progressed during peaceful times.

Fig. 6(b) and Fig. 6(c) show the transition of AID and SW, respectively. The blue and red lines mean the cases with and without pay-it-forward, respectively. According to these figures, AID and SW are larger in the case with pay-it-forward than in the case without pay-it-forward. AID with pay-it-forward is larger than that without pay-it-forward from the beginning of simulation until  $t = 3000$ . Meanwhile, SW show a

similar trend to AID, becoming larger in the case with pay-it-forward than that without pay-it-forward.

These results indicate that when the initial network has a significant degree disparity, pay-it-forward decreases ACC temporarily, but increases the degree of the players with smaller degrees (i.e., the degree disparity elimination effect). Accordingly, ACC and AID increase in the long run, resulting in the small-world property and higher SW.

#### 4. Discussion and Conclusions

In this study, we analyzed the long-term effects of pay-it-forward on social networks by conducting Monte Carlo simulations. The results showed that when the initial network was empty, pay-it-forward between people with different degrees who were not connected by daily interactions reduced degree disparity, bringing the small-world property to the network (**Fig. 3(a)** and **Fig. 3(b)**) and increasing social welfare (**Fig. 3(c)**). In the initial network with a large degree disparity, pay-it-forward reduced ACC temporarily, but in the long run, it increased ACC (**Fig. 6(a)**), bringing about the small-world property (**Fig. 6(a)** and **Fig. 6(b)**) and increasing social welfare (**Fig. 6(c)**). This was because the degree disparity was eliminated by reducing the number of players with smaller degrees (**Fig. 8**), facilitating link addition in peaceful times.

This study implies the positive sociological effect of pay-it-forward, which Atsumi [1] suggested, since it brings small-world property and increases social welfare. Its benefits to network formation were not necessarily short-term but gradual or long-term (i.e., a positive dynamic externality [16]). Moreover, when the initial network was empty, pay-it-forward had no negative effect on ACC and AID in the short run, but when the initial network had a significant degree disparity, it had a temporary negative effect on ACC. This may be because link rewiring leads cluster to dissolve in the existing network, as pointed out by Watts and Strogatz [8], and to accelerate the network formation during peaceful times, which requires a certain time for the positive sociological pay-it-forward effects to be realized.

In modeling pay-it-forward, we only considered the situation where only the recipients of the most recent disaster became support providers of the subsequent disaster. However, in reality, support from several past disaster areas has been reported [1]. Therefore, it is conceivable that more assistance will occur and more links will be formed among more people in reality than in this model. Accordingly, it can be said that this study has revealed the minimum impact of pay-it-forward on social networks.

By showing that pay-it-forward imparts small-world property to social networks, this study contributes to the significance of disaster relief volunteer organizations that activate pay-it-forward

and the rationale for promoting pay-it-forward. As described in Section 1, social networks with a small-world property realize that information and knowledge are transmitted quickly, trust is easily formed, and innovation emergence and productivity are expected to increase [13,14,15]. It is also noteworthy that while realizing small-world property, pay-it-forward eliminated social isolation and closed the disparity in social connection. Pay-it-forward and the disaster volunteerism activating it, can contribute to realizing such a society. Disaster NPOs play an important role in promoting volunteer activities during disasters [21], and they have encouraged residents of past disaster areas to participate in volunteer activities [1]. While these organizations have mainly focused on psychological effects, the aforementioned sociological effects can be the rationale for promoting pay-it-forward.

This study leaves several issues for future research. First, although the links were weighted equally in this model (i.e., unweighted network), the interactions formed through pay-it-forward may continue more rigidly than those formed through daily activities. In step 5 in Section 2.3, when a support provider forms a new link during a disaster, the probability of selecting a neighbor to delete the existing link is equal for all neighbors. On the other hand, it has been reported that emergent networks during a disaster can persist afterward [22] suggesting that interactions obtained through pay-it-forward are likely to be continued afterward. Therefore, it is necessary to incorporate this difference in weights into the model in the future: the weights of interactions formed through pay-it-forward and those obtained through peaceful times are different.

Second, pay-it-forward and network may co-evolve. Pay-it-forward spreads as more neighbors in the network engage in pay-it-forward behavior [11]. Thus, incorporating the increase in players who act altruistically during disasters into the network formation process is a future task.

Third, as shown in Section 2.3, this study assumed that link formation during disasters was determined probabilistically or independently of daily utility, but link formation during disasters may also occur under rational decision-making. As discussed in Section 1, the indebtedness gained from the assisted experience may lead to motivations for subsequent assistance activities [5,6]. By modeling such motives [23], it may be possible to present an integrated framework that can understand both the psychological and sociological aspects of pay-it-forward.

Although the above issues remain, this study is valuable as it demonstrated the sociological aspects of pay-it-forward (i.e., its effects on social networks) and elucidated its dynamics using a simple model. The most important result was to show that pay-it-forward brought about small-world property in society in the long term.

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