ECONOMIC BENEFIT EVALUATION OF RESERVATION SYSTEMS

Kyoto university ; Hishida kensuke

Reservation system

- In this study,
  a reservation system is regarded as a mechanism,
  which allocates the services to potential customers
  along the first-comer-first-served principle.

- Key wards
  [revelation mechanism, real option,
  economic benefits, monopolistic market]

Motivation

- I want to know in this study

1,what kind of influence the reservation system has on the consumers surplus, the company benefit, and the public welfare.

2,necessity of the fare regulation.

The benefits of the reservation system

Option to purchase

- Reservation system supplies the customers an option to purchase certainly the service in the future.

Revelation mechanism

- Revelation mechanism attaches the priority to those who derive more satisfaction from the service than others along the first-comer-first-served principle.

The preconditions of this model

- Monopolistic market
- A single service market.
- This model has 1st and 2nd term.
- There is limitation to a supply.
- There are two kinds of consumers.

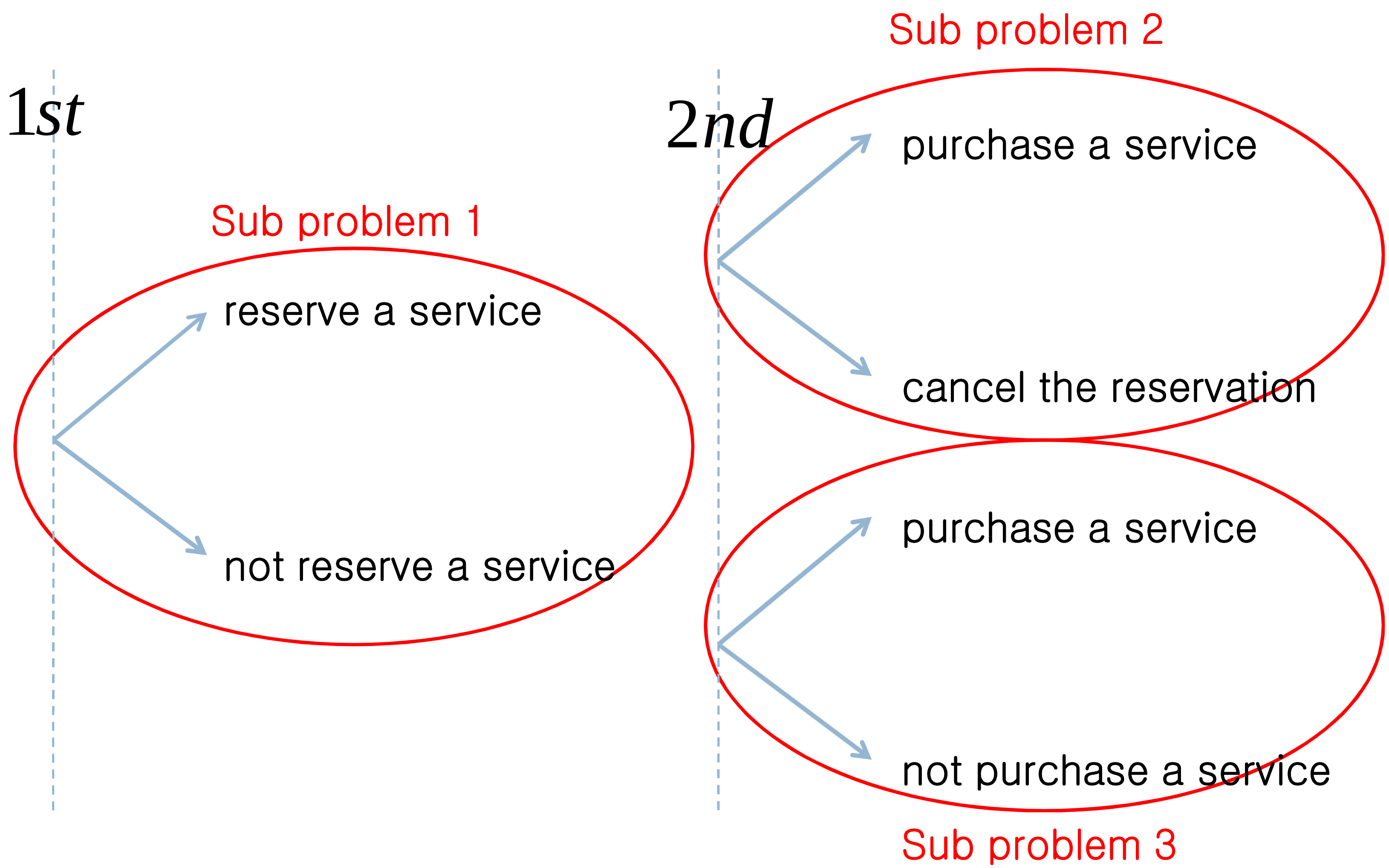
Type H ⇔ Type L

Type H ⇔ Type L

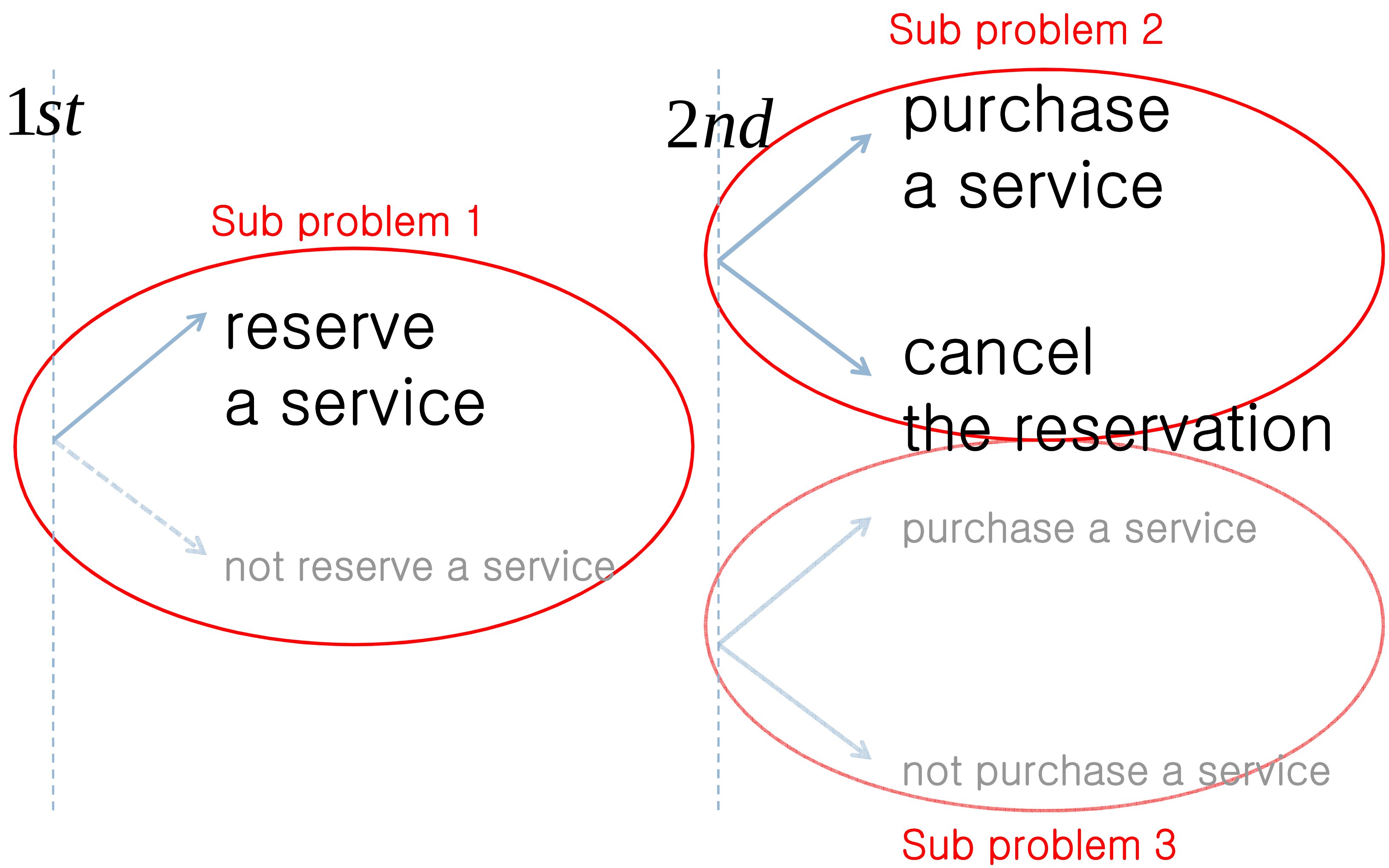
- Type H is the customers who get higher utility from the service than Type L and reserve at 1st term.

- Type L is the customers who get lower utility from the service than Type H and don’t reserve at 1st term.

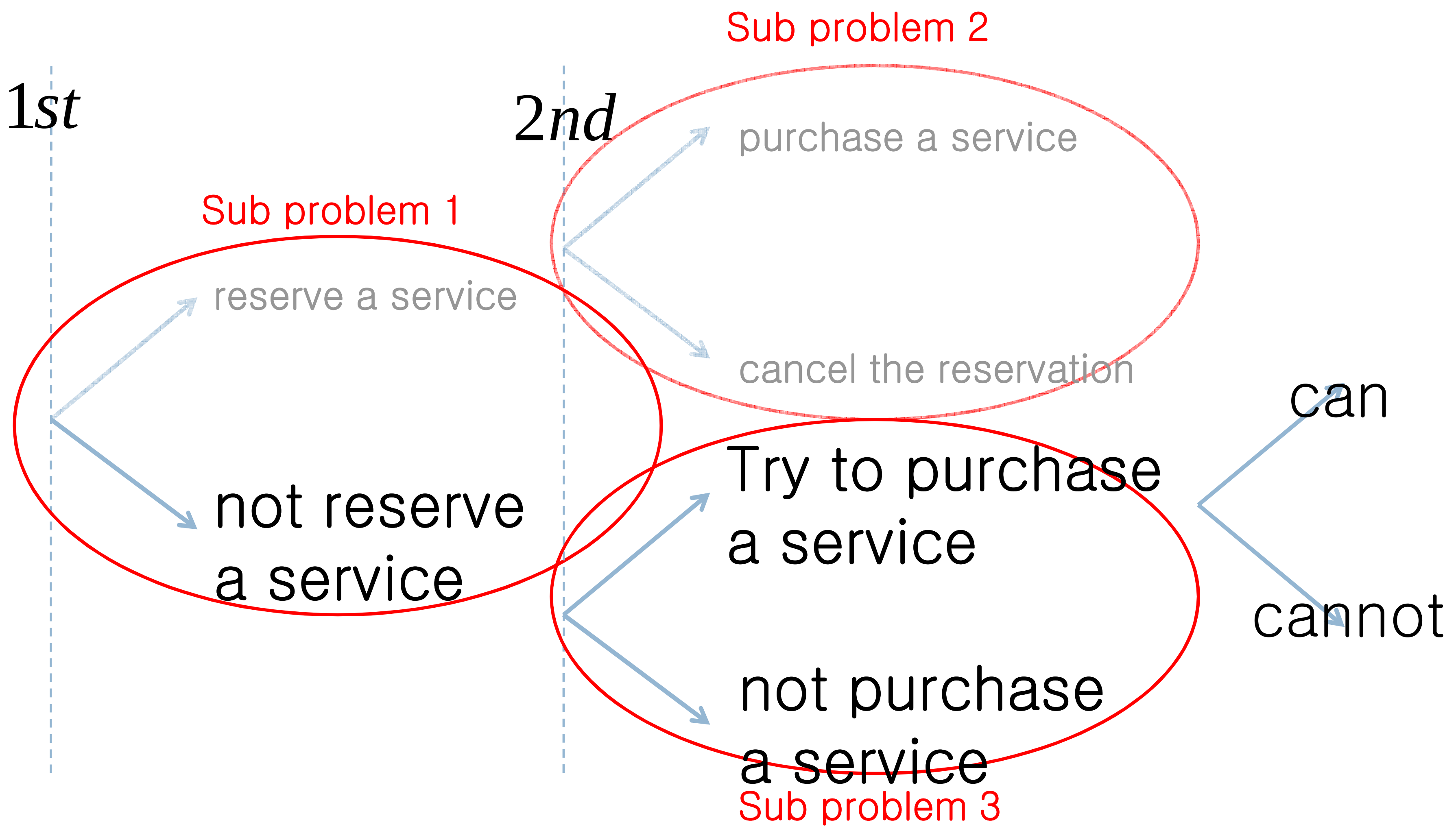
The structure of consumer's decision to reserve and purchase in this model



The structure of Type H's decision to reserve and purchase in this model



The structure of Type L's decision to reserve and purchase in this model



The setting of this model ①

- The utility that Type H gets by the purchase of the service

$$u_H$$

- The utility that Type L gets by the purchase of the service

$$u_L \qquad (< u_H)$$

- The number of Type H

$$1$$

- The number of Type L

$$Q$$

The setting of this model ②

- The fare to pay when consumers purchase a service

$$p$$

- The fare to pay when consumers canceled a service

$$c \qquad (< p)$$

- The utility to get

when consumers don't purchase a service

$$\varepsilon = \begin{cases} 0 & \text{in probability } q \\ \bar{\varepsilon} & \text{in probability } 1-q \end{cases}$$

The behavior of Type H

1)If Type H reserved a service at 1st term,

- The utility to get

  when Type H purchase a service at 2nd term

$$V_H = u_H - p$$

- The utility to get

  when Type H canceled a service at 1st term

$$V_H = \varepsilon - c = \begin{cases} 0 - c \text{ in probability } q \\ \bar{\varepsilon} - c \text{ in probability } 1 - q \end{cases}$$

The behavior of Type H

If it is $0 - c < u_H - p < \bar{\varepsilon} - c < \bar{\varepsilon}$,

- The utility to get when Type H reserved at 1st

$$V_H = \begin{cases} u_H - p & \text{When } \varepsilon = 0 \text{ in probability q} \\ \bar{\varepsilon} - c & \text{When } \varepsilon = \bar{\varepsilon} \text{ in probability 1-q} \end{cases}$$

When we purchase the service

When we <u>don't purchase</u> the service
(cancel)

The behavior of Type H

2)If Type H didn't reserve a service at 1st,

- The utility to get

  when Type H purchase a service at 2nd

$$V_H = u_H - p$$

- The utility to get

  when Type H don't purchase a service at 2nd

$$V_H = \varepsilon = \begin{cases} 0 & \text{in probability } q \\ \bar{\varepsilon} & \text{in probability } 1-q \end{cases}$$

The purchasability probability

- When consumers didn't reserve at 1st term, there is those who cannot purchase at 2nd term. We can set purchasability probability in h .

1,Probability to try to purchase and get it

$$qh$$

2,Probability to try to purchase but don't get it

$$q(1-h)$$

The behavior of Type H

$$0 - c < u_H - p < \bar{\varepsilon} - c < \bar{\varepsilon}$$

- The utility to get when Type H didn't reserve at 1st term

$$V_H = \begin{cases} u_H - p & \text{When consumers can get in probability qh} \\ 0 & \text{When consumers cannot in probability q(1−h)} \\ \bar{\varepsilon} & \text{When } \varepsilon = \bar{\varepsilon} \text{ in probability 1−q} \end{cases}$$

In summarizing of Type H's behavior

If Type H did make a reservation

Type H get the utility

$$V_H = \begin{cases} u_H - p & in\ prob..q \\ \bar{\varepsilon} - c & in\ prob..1-q \end{cases}$$

If Type H didn't make a reservation

Type H get the utility

$$U_H = \begin{cases} u_H - p & in\ prob..qh \\ 0 & in\ prob..q(1-h) \\ \bar{\varepsilon} & in\ prob..1-q \end{cases}$$

Expected utility of Type H

1)If we reserve a service at 1st term,
we get expected utility EV_H .

$$EV_H = q(u_H - p) + (1-q)(\bar{\varepsilon} - c)$$

2)If we don't reserve a service at 1st term,
we get expected utility EU_H.

$$EU_H = qh(u_H - p) + q(1-h)(0) + (1-q)\bar{\varepsilon}$$
$$= qh(u_H - p) + (1-q)\bar{\varepsilon}$$

3)Whether to reserve or not.

3)Whether to reserve or not.

$$EV_H \geq EU_H \Rightarrow \text{Type H reserve}$$

$$EV_H < EU_H \Rightarrow \text{Type H don't reserve}$$

comment

WE can describe the expected utility of type L
in the same manner of Type H.

Expected utility of Type L

1)If we reserve a service at t=0,
we get expected utility EV_L .

$$EV_L = q(u_L - p) + (1-q)(\bar{\varepsilon} - c)$$

2)If we don't reserve a service at t=0,
we get expected utility EU_L .

$$EU_L = qh(u_L - p) + (1-q)\bar{\varepsilon}$$

3)Whether to reserve or not.

$EV_L \geq EU_L \Rightarrow$ Type L reserve

$EV_L < EU_L \Rightarrow$ Type L don't reserve

The purchasability probability

- Type H reserves and Type L doesn't reserve.
  So, there is a possibility that just Type L cannot purchase the service.
  Type H purchases it in probability q.
  The number of Type H is 1, so q people of them purchase it. Then just $1-q$ services are left in the market for Type L.
  In other words, the supply for type L is $1-q$ services.
  And the demand of Type L is qQ.
  Therefore the purchasbility probability is as follows,

The purchasability probability

$$h = \frac{1-q}{qQ} = \frac{supply\ \ for\ Type\ L}{demand\ of\ Type\ L}$$

The next condition is necessary to give the system a meaning.

$$1-q < qQ$$

$$supply\ \ for\ \ Type\ L < demand\ of\ Type\ L$$

The constraints for revelation selection

$EV_H \geq EU_H \Rightarrow$ Type H consumers reserve

and

$EV_L < EU_L \Rightarrow$ Type L consumers do not reserve

$$q(u_H - p) + (1-q)(\bar{\varepsilon} - c) \geq qh(u_H - p) + (1-q)\bar{\varepsilon}$$

and

$$q(u_L - p) + (1-q)(\bar{\varepsilon} - c) < qh(u_L - p) + (1-q)\bar{\varepsilon}$$

and

$$u_L \geq p$$

More simply

The revelation selection constraints

$$q(1-h)(u_H - p) \geq (1-q)c \quad \cdots sc1$$

and

$$q(1-h)(u_L - p) < (1-q)c \quad \cdots sc2$$

and

$$u_L \geq p \quad \cdots sc3$$

The expectation of the utility that we cannot get by cannot purchasing service when I did not make reservations

The expectation of the amount of loss by canceling it when we made reservations

When the constraints are satisfied,

Type H reserve a service at 1st term ,
Type L don't reserve a service at 1st term
and
Type L try to purchase a service at 2nd term.

Then,
the self-selection mechanism does work!

Profit maximization problem ①

$$\max_{p} p$$

Subject to

$$q(1-h)(u_H - p) - (1-q)c \geq 0 \quad \cdots \text{sc1}$$

and

$$q(1-h)(u_L - p) - (1-q)c < 0 \quad \cdots \text{sc2}$$

and

$$u_L \geq p \quad \cdots \text{sc3}$$

The strongest limitation
In these constraints on p

$$p^* = u_L$$

comment

$p^* = u_L$ satisfies

$$q(1-h)(u_H - p) - (1-q)c < 0 \quad \cdots \text{sc2}$$

$$u_L \geq p \quad \cdots \text{sc3}$$

Profit maximization problem ②

$$\max_{c} c$$

Subject to

$$q(1-h)(u_L-p)-(1-q)c \geq 0 \Leftrightarrow \frac{q(1-h)}{(1-q)}(u_L-p) \geq c$$

…sc1

$$c^* = \frac{q(1-h)}{1-q}(u_H-u_L)$$

The fares in reservation equilibrium

The fares in equilibrium

$$\begin{cases} p^* = u_L \\ c^* = \dfrac{q(1-h)}{1-q}\delta \end{cases}$$

A notice matter: $\delta = u_H - u_L$

The economic public welfare

$$Total \quad expecred \quad customer \quad surplus \quad of \quad Type\,H\ EW_H$$

$$EW_H = Y + EV_H$$

$$= Y + q(u_H - p) + (1-q)(\bar{\varepsilon} - c)$$

$$Total \quad expecred \quad customer \quad surplus \quad of \quad Type\,L\ EW_L$$

$$EW_L = Y + EU_L$$

$$= QY + Q\{qh(u_L - p) + q(1-h)(0) + (1-q)\bar{\varepsilon}\}$$

$$The \quad profit \quad of \quad the \quad company \quad \pi$$

$$\pi = p + (1-q)c - F$$

$$The \quad total \quad public \quad surplus \quad SW$$

$$SW = (1+Q)Y + q(u_H - p) + qQh(u_L - p) + (1-q)(1+Q)\bar{\varepsilon} + p - F$$

The fares in reservation equilibrium

The fares in equilibrium

$$\begin{cases} p^* = u_L \\ c^* = \dfrac{q(1-h)}{1-q}\delta \end{cases}$$

A notice matter: $\delta = u_H - u_L$

The economic public welfare in the reservation equilibrium

Total expecred customer surplus of Type H EW_H^*

$$EW_H^* = Y + EV_H^*$$
$$= Y + qh(u_H - u_L) + (1-q)\bar{\varepsilon}$$

Total expecred customer surplus of Type L EW_L^*

$$EW_L^* = Y + EU_L^*$$
$$= Y + (1-q)Q\bar{\varepsilon}$$

The profit of the company π^*

$$\pi^* = u_L + (1-q)c - F$$

The total public surplus SW^*

$$SW^* = (1+Q)Y + q(u_H - u_L) + (1-q)(1+Q)\bar{\varepsilon} + p - F$$

Supplement

$$
\begin{aligned}
EW_H^* &= Y + EV_H^* \\
&= Y + q(u_H - p) + (1-q)(\bar{\varepsilon} - c) \\
&= Y + q(u_H - p) + (1-q)\bar{\varepsilon} - (1-q)c \\
&= Y + q(u_H - u_L) + (1-q)\bar{\varepsilon} - (1-q)\frac{q(1-h)}{(1-q)}(u_H - u_L) \\
&= Y + \cancel{q(u_H - u_L)} + (1-q)\bar{\varepsilon} \cancel{- q(u_H - u_L)} + qh(u_H - u_L) \\
&= Y + qh(u_H - u_L) + (1-q)\bar{\varepsilon}
\end{aligned}
$$

the standard equilibrium

*The purchasability probability
in the standard equilibrium

$$h^{\circ} = \frac{1}{q(1+Q)} = \frac{supply\ for\ Type\ H\ and\ Type\ L}{demand\ of\ Type\ H\ and\ Type\ L}$$

*The fare in the standard equilibrium

$$p^{\circ} = u_L (= p^{*})$$

The economic public welfare in the standard equilibrium

$Total \quad expecred \quad customer \quad surplus \quad of \quad Type\,H\; EW_H^\circ$

$$EW_H^\circ = Y + qh^\circ(u_H - u_L) + (1-q)\bar{\varepsilon}$$

$Total \quad expecred \quad customer \quad surplus \quad of \quad Type\,L\; EW_L^\circ$

$$\begin{aligned} EW_L^\circ &= Y + EV_L \\ &= Y + (1-q)Q\bar{\varepsilon} \end{aligned}$$

$The \quad profit \quad of \quad the \quad company \quad \pi^\circ$

$$\pi^\circ = u_L - F$$

$The \quad total \quad public \quad surplus \quad SW^\circ$

$$SW^\circ = (1+Q)Y + qh^\circ(u_H - p) + (1-q)(1+Q)\bar{\varepsilon} + u_L - F$$

The economic effect of the reservation system

The subject	Reservation equilibrium	Standard equilibrium	The effect
Type H	$Y+qh\delta+(1-q)\bar{\varepsilon}$	$Y+qh^{\circ}\delta+(1-q)\bar{\varepsilon}$	$(h-h^{\circ})q\delta$
Type L	$QY+(1-q)Q\bar{\varepsilon}$	$QY+(1-q)Q\bar{\varepsilon}$	0
company	$u_L+(1-q)c^{*}-F$	u_L-F	$(1-q)c^{*}$
Social surplus			$(1-h^{\circ})q\delta$

$$\delta=u_H-u_L$$

The analysis of the effect to Type H

The effect to Type H ： $\Delta_H = (h - h^{\circ})q\delta$

$$\Delta_H = \left(\frac{1 - qQ - q}{Q(1+Q)}\right)\delta < 0$$

Worse off

Considering following limitation conditions

$$h = \frac{1-q}{qQ}, h^{\circ} = \frac{1}{q(1+Q)}, 1 - q < qQ$$

The analysis of the effect to the other objects

The effect to Type L : $\Delta_L = 0$

The effect to company : $\Delta_\pi = (1-q)c^*$

$$\Delta_L = 0$$

$$\Delta_\pi \geq 0$$

Better off

The analysis of the effect to social surplus

The effect to social surplus ：

$$\Delta_{SW} = (1 - h^{\circ})q\delta$$

$$\Delta_{SW} = \left(\frac{qQ + q - 1}{1 + Q}\right)\delta > 0$$

Better off

Considering following limitation conditions

$$h^{\circ} = \frac{1}{q(1+Q)}, 1 - q < qQ$$

The economic effect of the reservation system

The subject	The effect
Type H	$(h-h^{\circ})q\delta=(1-h^{\circ})q\delta-(1-q)c^{*}<0$
Type L	0
company	$(1-q)c^{*}\geq 0$
Social surplus	$(1-h^{\circ})q\delta>0$

From this list,
We can find that cancel fare moves the income from Type H to the company.