Price Coordination Mechanism in a Shopping Area with Demand Externality (Draft version)*

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Abstract

In this paper, a linear market model is proposed to investigate the horizontal price competition among multiple shops in a shopping area and a shopping center. The price of a commodity sold at one shop in the shopping area may affect the commodity demand sold in another shop located in the same area. We call such an externality the 'demand externality'. Shops in the shopping area fail to internalize the demand externality because they set prices of commodities in a decentralized way. We have theoretically pointed out that, due to such a demand externality, the shops in the shopping area enjoy a relatively smaller market size than that of the shopping center who can determine the prices for multiple commodities in a centralized way. The equilibrium in this model results in inefficient market structure in terms of the total transportation costs in an economy. Moreover, the paper shows that shops in the shopping area can coordinate their pricing strategies and expand their market size by issuing discount tickets to the customers, by which the respective shops in the shopping area can internalize the demand externalities.

keywords: *price coorination, demand externality, discount system, community governance*

1 Introduction

In Japan the Central City Invigoration Law was established in 1998 along with many efforts to revitalize the center of cities. It may be impossible however, to restrain the decline of the central areas of the cities[1]. In this paper, we point out the mechanism that causes the decline in a shopping area (hereinafter called SA) is due to the failure of shops to coordinate their pricing strategy of the commodities that they sell in their shops. Consumers often choose to shop in a commercial setting under the condition so called 'multipurpose shopping'. In such a situation, they will purchase some commodities in one shop. If the kind of commodities sold in both commercial settings are identical, comsumers will choose one commercial setting (SC or SA) considering retail prices of commodities they want to purchase and the trip costs. In this case, price of one shop in the SA may affect the demand for a commodity which is sold in another shop located in the same area. We call such an externality 'demand externality'. Shops in the SA fail to internalize the demand externality because they set prices of commodities in a decentralized way while SC can internalize the demand externality in a centralized way. The equilibrium in this model results in an inefficient market structure in terms of total transportation costs of an economy.

In a realistic situation, SAs and SCs are located discretely and they have distinct market size. The failure to coordinate the price strategy in SA may cause the decline of SA. However we don't mean that the decline of SA itselt disturbs the social efficiency. The decline of SA would rather be regarded as a problem where SA cannot realize the efficient market size because of the demand externality. Heavy concentration of comsumers to the SC will generate excessive trip costs and social costs like traffic jams near the SC or latency time for parking. From a standpoint of social efficiency, it is important for social optimum that shops in the SA can coordinate their price strategy then inter-

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nalize the demand externality.

In Japan we can find many SAs introducing discount systems, for example Reward Card systems, in which SA makes a discount when consumers collect some points. We show in this paper that the SA can coordinate their pricing strategies and expand its market size by issuing a kind of discount tickets to the customers, by which the respective shops in the shopping area can internalize the demand externalities. Moreover, we investigate the formation of the club-like organization and its stability.

This paper is organized as follows. Section 2 presents the fundamental ideas of the paper. In section 3, the basic model is formulated. The model with discount ticket system is then proposed in section 4. Finally, section 5 concludes the paper.

2 Fundamentals

2.1 Review of Related Literature

Some people point out that recently shopping behavior of consumers is shifting from SA located in central areas of the city to SC in the suburbs and this may cause the decline in SA. Many shops in declining shopping area have no choice but to go out of business and thus creating so called 'shuttered street' extensively. Such a mechanism is studied by Modani[2].

Eaton and Lipsey[3] explained the mechanism of commercial accumulation, focusing on the fact that consumers often purchase some commodities in one time shopping, 'multipurpose shopping'. Their finding is that economies of scale would be prominent in multipurpose shopping. Consumers can save a great deal of time by purchasing commodities in one place, instead of many places, so called 'one-stop shopping'[4].

In Eaton and Lipsey's model (EL model), they assume that the purchase frequency of comsumers is constant and retail prices are determined exogenously in order to describe the mechanism of the effect of one-stop shopping which may bring about the commercial accumulation. In this paper, we also focus on the one-stop shopping and the pricing strategy of commdercial settings and we assume that commercial accumulation is given whereas EL model employ the endogenous model which explains commercial accumulation.

Some research about inter-store externalities can be founded in the literrature[17, 18, 19, 20, 21]. Brueckner analyzed the optimum allocation problem in SC with inter-store externalities[17]. In a SC with inter-store externalities, a developer sets rental rates for tenants and allocates each shop to the empty areas of the SC in order to maximize the profit of the developer. Miceli and Sirmans[18] formulated the problem of the optimum allocation in SC with inter-store externalities as a common agency problem and pointed out this problem could be caused by the developers' underinvestment. We can sum up the research as below. Space allocation can be coordinated by the developer setting optimum rental rate to maximize the profit of SC because it is assumed that the centralized decision maker can coordinate resource allocation with inter-store externalities by setting the optimum price for the resource.

Goto et al.[6] revealed the processes that an existing shopping area and shopping center in the suburbs gets each market when their location is given. In this research a commercial system consisting of a shopping area located in the city centre and a shopping center located in the suburb is assumued. Under this situation, it is shown that the market equilibrium of the system cannot realize the optimum market size at all times, but it can by changing the parking fee. Other empirical analyses about shopping behavior are also developed[7, 8, 9].

In this paper, we focus on the discount ticket system as a tool that realizes the coordination of the pricing strategy in an existing shopping area. We can find a paper that describes the rock-in effect of the discount ticket system[10]. The rock-in effect means that in the case that consumers continually purchase a particular commodity and the shops that provide the commodity will offer services like discount to the customers. Consumers cannnot change to other commodity or services easily because of increasing of switching costs. In this paper, we focus on the function of the discount ticket system to coordinate the pricing strategy in SA, therefore we don't consider the consumers' continual shopping behavior.

2.2 Consumers' Multipurpose Shopping Behavior and Demand Externality

Consumers often purchase some commodities in one time shopping. Especially, when they go shopping and purchase some daily commodities, they can bring them back home if they go there by car. We define this kind of behavior as 'multipurpose shopping'. Then consumers will select one commercial setting, considering trip cost and the total of the price they will pay. Under such a situation, the price of a commodity will affect not only the demand of the commodity but also the consumers' choice of the commercial setting. The pricing strategy of a commoditiy in the commercial architecture, as a result, may affect the demand of another commodity in the same place. Consider an example. In a shopping area consisting of a number of shops, a shop which bundles one commodity makes a discount. In such a situation, consumers who buy the discounted commodity in the SA will prefer to purchase the same commodity rather than to purchase in the other commercial setting which makes a discount. As a result, the number of consumers who purchase the commodity in SA will increase while the number of consumers who go to other commercial setting for the commodities which is not discounted will decrease. That is to say, if a shop in the SA make a discount and consumers' behavior shift from SC to SA, the discount may contribute to the increase of the demand of other commodities which are not discounted. We call this effect 'Demand Externality'.

In general, a shop in a shopping area can handle less kind of commodities compared with a SC because a shop in a shopping area is constrained by the size of shop or technical problems, for example. This is the reason why the prices of commodities in SA are determined in a decentralized way. When each shop in the SA set the price of the commodities individually, they cannot internalize such a demand externality as we mentioned above, hence the SA is not able to realize the pricing strategy to maximize the profit of SA as a whole because each shop in SA cannot enjoy the effect caused by making a discount if each shop behaves individually. On the other hand, in SC, one decision maker handles multiple commodities and can set the prices of commodities in a centralized way. Therefore SC determine its pricing strategy considering comsumers' choice of commercial architecture whereas a shop in the SA considering only the demand of the commodity which they sell.

2.3 Framework

In this study, we also focus on SC owned and managed as a unit by a developer[4]. A shopping area sometimes makes a joint offering or some events but these are not able to function as a contract like SC. Therefore we can suppose that shops in the SA behave individually. On the other hand, SC confirms its rights and duties by making a contract to maximize its joint profit[4]. In other words, a SC have a centralized decision-making mechanism to maximize its profit as a whole whereas a SA have only a decentralized decision-making mechanism where each shop make a decision individually. Of course we can consider the diversity of goods sold in commercial architecture or shop size as the differences between the SA and the SC. In this research, however, we presuppose that all conditions except the decision-making mechanism between the SA and the SC are the same because we try to analyze the economic consequences induced by the mechanism.

3 Basic Model

3.1 Assumptions

Suppose that a linear spatial market defined in $\Theta = \{\theta | \theta \in [0, 1]\}$ as shown in **Figure.1** and consumers who have homogeneous preference are distributed equally between the SA and the SC. The density is $N(N-1)/2$. Consumers demand 2 kinds of commodities out of N kinds of commodities sold in commercial setting and obtain utility by buying the commodities that they choose. In the basic model, we assume that consumers choose the commodities randomly. The existing shopping area is located in $\theta = 0$ and consisting of N shops which sell N kinds of commodities individualy. The spatial scale of the shopping area is vanishingly small. The SC which can handle N kinds of commodities is located on $\theta = 1$. We do not consider any condition that could encourage newcomers. Consumers in the linear spatial market make a trip to

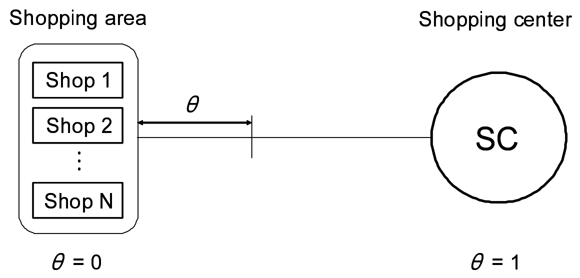


Figure.1: A Linear Spatial Market

the SA or the SC and purchase the commodities that they need. Commodities G_i $(i = 1, \cdots, N)$ are indivisible goods. We employ the f.o.b. mill pricing model[5] where each shop cannot differentiate the price of goods and consumers defray the retail prices (mill price) and trip costs. All commodities sold in the SA and the SC are homogeneous. Supose that the circulation trading market is perfectly competitive and cost prices in the SA and the SC are identical. In this paper, we purposely neglect the economics of scale in technology and fixed costs to maintain the shops.

3.2 Model Formulation of Consumers

Let us consider a situation in which consumers purchase G_i and G_j, where $i \neq j$ $(i, j = 1, \cdots, N)$. The utility which consumers can get by consuming G_i and G_j is denoted by

$$u(i, j) = u_i + u_j \tag{1}$$

Suppose that the utilities of G_n $(n = 1, \cdots, N)$ are identical for consumers. Therefore in such a situation, we can get the formulation as follows;

$$u(i, j) = 2\overline{u} \tag{2}$$

Now focus on the combination of any 2 goods out of N kinds of goods (hereinafter called mix goods), which is formulated as

$$\Lambda = \{(i, j) | i \neq j; i, j = 1, \cdots, N\} \tag{3}$$

Let us assume the probability that any mix goods $(i, j) \in \Lambda$ are chosen out of Λ is identical to all combinations and this probability occurs independently. Therefore we can define the probability $\Pr[(i, j) \in \Lambda]$ that any mix goods $(i, j) \in \Lambda$ is chosen as;

$$\Pr[(i, j) \in \Lambda] = \frac{1}{{}_N C_2} = \frac{2}{N(N-1)}, \quad \forall\, i, j \tag{4}$$

Consumers' utilities depend on the prices of goods and trip costs. Transpotation costs for the trip are 1 unit of money per unit distance. Consumers select commercial setting and go to the SA or the SC to maximize their utility. We define $V^s(\theta)$ as indirect utility that consumer can get by purchasing the commodities in the SA and $V^l(\theta)$ in the SC. In such a situation, indirect utility function of consumer H_θ located on θ with $(i, j) \in A$ can be expressed as follows;

$$V^s(i, j : \theta) = u_i + u_j - p_i^s - p_j^s + Y - \theta \quad \text{(shopping in SA)} \tag{5}$$

$$V^l(i, j : \theta) = u_i + u_j - p_i^l - p_j^l + Y - (1 - \theta) \quad \text{(shopping in SC)} \tag{6}$$

, where Y is income of consumer, p_k^s is the retail price of commodity G_k sold in SA, p_k^l is the retail price of G_k sold in SC, and suffix s and l means the SA and the SC. Now consider a situation where all goods are symmetrical. This condition yields

$$p_i^s = \overline{p}^s \qquad (i = 1 \cdots, N) \tag{7}$$

$$p_i^l = \overline{p}^l \qquad (i = 1 \cdots, N) \tag{8}$$

By substituting (7) and (8) to (5) and (6), we can get

$$V^s(i, j : \theta) = 2\overline{u} - 2\overline{p}^s + Y - \theta \quad \text{(shopping in SA)} \tag{9}$$

$$V^l(i.j : \theta) = 2\overline{u} - 2\overline{p}^l + Y - (1 - \theta) \quad \text{(shopping in SC)} \tag{10}$$

Now let us assume that the following formulations are approved.

$$\overline{u} - \overline{p}^s \geq 1 \tag{11}$$

$$\overline{u} - \overline{p}^l \geq 1 \tag{12}$$

In other words, the above conditions assure that all consumers in the linear spatial market make a trip to either commercial setting and purshase 2 kinds of commodities according to their demand. Observing the retail prices vector $\boldsymbol{p} = (p_1^s, \cdots, p_N^s, p_1^l, \cdots, p_N^l)$, consumer H_θ make a trip to the SA and buy commodities if $V^s(\theta) \geq V^l(\theta)$ whereas Consumer H_θ choose the SC if $V^s(\theta) < V^l(\theta)$. Given $(i,j) \in A$, there exists a point where comsumers can obtain the same utility by buying commodities even in either commercial setting since $V^s(\theta)$ is monotonically decreasing for θ. Hence we have next formulation,

$$V^s(r_{ij}(\boldsymbol{p})) = V^l(r_{ij}(\boldsymbol{p})) \tag{13}$$

where $r^s_{ij}(\boldsymbol{p})$ is a bifurcation point of the market. This definition gives;

$$\begin{aligned} r_{ij}(\boldsymbol{p}) &= \frac{(p^l_i + p^l_j) - (p^s_i + p^s_j) + 1}{2} \\ &= \overline{p}^l - \overline{p}^s + \frac{1}{2} \end{aligned} \tag{14}$$

In what follows, we define $r_{ij}(\boldsymbol{p})$ as $\overline{r}$ in order to simplify the argument, thus we can express the shopping behavior of consumer with mix goods $(i,j) \in \Lambda$ as;

$$\begin{cases} \text{comsumers go to the SA} & \text{if} \quad \theta \leq \overline{r} \\ \text{comsumers go to the SC} & \text{if} \quad \theta > \overline{r} \end{cases} \tag{15}$$

3.3 Social Optimum Model

In this section, we employ the Hotelling-Smithies competition[14] where each player decides pricing strategy, considering other players' pricing strategies as given variables. Suppose that the scale of the SC is so much larger than that of the SA then the retail prices of the SC are determined by other factors except the scale. In such a situation, pricing strategy of the SA doesn't affect that of SC. Each shop in the SA sets its pricing strategies, assuming prices in the SC are given. Hence the retail price of goods G_i sold in the SC is expressed as p^l_i. A shop S_i in the SA decides the price p^s_i of goods G_i. The expected profit function of S_i $(i = 1, \cdots, N)$ is described as follos;

$$\begin{aligned} \pi^s_i &= \frac{2}{N(N-1)} \sum_{j \neq i} (p^s_i - w_i) \int_0^{\overline{r}} \frac{N(N-1)}{2} d\theta \\ &= \sum_{j \neq i} (p^s_i - w_i)\overline{r} \end{aligned} \tag{16}$$

we can formulate the profit maximization problem of the SA as below;

$$\max_{\boldsymbol{p}^s} \Pi^s \tag{17}$$

where

$$\Pi^s = \sum_{i=1}^{N} \pi^s_i \tag{18}$$

so we get the first order optimization condition of this problem as the following formulation

$$\begin{aligned} \frac{\partial \Pi^s}{\partial p^s_i} &= \sum_{j \neq i} \overline{r} - \frac{N-1}{2}(p^s_i - w_i) \\ &\quad - \frac{1}{2} \sum_{j \neq i} (p^s_j - w_j) = 0 \qquad (i = 1, \cdots, N) \end{aligned} \tag{19}$$

, where the first part of right-hand side of this formulation means the direct profit change caused by the price change, the second part illustrates the indirect profit change caused by the demand change of a goods, which is affected by the change of pricing strategy of the goods and the third part represents the profit change caused by the demand change of 'other' goods in the same area, which is affected by the price change of the price of a goods. Equation (19) can be reformulated as;

$$\begin{aligned} &\frac{N-1}{2}(p^l_i - p^s_i + 1) + \frac{\sum_{j \neq i}(p^l_j - p^s_j)}{2} \\ &\quad - \frac{N-1}{2}(p^s_i - w_i) - \frac{\sum_{j \neq i}(p^s_j - w_j)}{2} = 0 \\ &\qquad (i = 1, \cdots, N) \end{aligned} \tag{20}$$

we figure out a sum of (35) with all i and get the next formulation

$$P^{s\circ} = \frac{P^l + W}{2} + \frac{N}{4} \tag{21}$$

where

$$P^s = \sum_{i=1}^{N} p^s_i \tag{22}$$

$$P^l = \sum_{i=1}^{N} p^l_i \tag{23}$$

In a similar way we can get the expected profit function of the SC as follows

$$\begin{aligned} \Pi^l &= \sum_{i=1}^{N} \frac{2}{N(N-1)} \sum_{j \neq i} (p^l_i - w_i) \int_{\overline{r}}^{1} \frac{N(N-1)}{2} d\theta \\ &= \sum_{i=1}^{N} \sum_{j \neq i} (p^l_i - w_i)(1 - \overline{r}) \end{aligned} \tag{24}$$

We can get the profit maximization problem of the SA as

$$\max_{\boldsymbol{p}^l} \Pi^l \tag{25}$$

Then the first order optimazation problem can be expressed as

$$\frac{N-1}{2}(p_i^s - p_i^l + 1) + \frac{\sum_{j\neq i}(p_j^s - p_j^l)}{2} - \frac{N-1}{2}(p_i^l - w_i) - \frac{\sum_{j\neq i}(p_j^l - w_j)}{2} = 0 \quad (i = 1, \cdots, N) \tag{26}$$

we figure out a sum of (35) with all i and get the next formulation

$$P^{l\circ} = \frac{P^s + W}{2} + \frac{N}{4} \tag{27}$$

Finally, we have the Nash Equiliblium solution in this model by solving (21) and (27) as below

$$P^{l\circ} = P^{s\circ} = W + \frac{1}{2}N \tag{28}$$

Under this situation, the bifurcation point of the market can be calculated as

$$\overline{r}^{\circ} = \frac{1}{2} \tag{29}$$

, which realize the optimum market size because if $r_{ij}^{\circ}(\boldsymbol{p}) = 1/2$ the total trip costs consumers have to pay will be minimum.

3.4 Decentralized Decision Making Model

In this section, we formulate the profit maximization problem of the SA with a decentralized way. This problem can be expressed as below;

$$\max_{p_i^s} \pi_i^s \qquad (i = 1, \cdots, N) \tag{30}$$

The first order optimization condition of this model is following;

$$\frac{\partial \pi_i^s}{\partial p_i^s} = \sum_{j\neq i} \overline{r} - \frac{N-1}{2}(p_i^s - w_i) = 0 \quad (i = 1, \cdots, N) \tag{31}$$

Comparing (19) to (31) we can find that this formulation lacks what is equivalent to the third part of right-hand side of equation (19). This means that each shop in the SA doesn't consider the demand externality when they set their pricing strategies in the decentralized way. Equation (31) can be reformulated as follows;

$$\frac{N-1}{2}(p_i^l - p_i^s + 1) + \frac{\sum_{j\neq i}(p_j^l - p_j^s)}{2} - \frac{N-1}{2}(p_i^s - w_i) = 0 \qquad (i = 1, \cdots, N) \tag{32}$$

we calculate a sum of (32) with all i and get the next formulation.

$$P^{s*} = \frac{2P^l + W + N}{3} \tag{33}$$

Since the SC's behavior can be written in the same way in social optimum model, we have

$$\max_{\boldsymbol{p}^l} \Pi^l \tag{34}$$

The first order optimazation problem can be expressed as

$$\frac{N-1}{2}(p_i^s - p_i^l + 1) + \frac{\sum_{j\neq i}(p_j^s - p_j^l)}{2} - \frac{N-1}{2}(p_i^l - w_i) - \frac{\sum_{j\neq i}(p_j^l - w_j)}{2} = 0 \quad (i = 1, \cdots, N) \tag{35}$$

Thus we obtain the following equation

$$P^{l*} = \frac{P^s + W}{2} + \frac{N}{4} \tag{36}$$

Finally, we have the Nash Equiliblium solution in this model by solving (33) and (35) as below

$$P^{s*} = W + \frac{3}{4}N$$
$$P^{l*} = W + \frac{5}{8}N \tag{37}$$

Under this situation, the bifurcation point of the market can be calculated as

$$\overline{r}^{*} = \frac{3}{8} \tag{38}$$

At variance with social optimum model, a efficient market cannot be realized in this model where each shop in the SA decides their pricing strategies in a decentralized way. It is obvious that the next equation should be approved as below

$$\Pi^{s\circ} > \Pi^{s*} \tag{39}$$

so we can obtain the following **proposition 1**.

proposition 1: When we assume consumers' multi-purpose shopping behavior, each shop in the SA sets their pricing strategies in a decentralized way and thus fails to internalize demand externality. As a result, profit which each shop in the SA can enjoy is smaller than that of the SC that determines their retail prices in a centralized way.

4 Model with Discount Ticket System

4.1 Assumptions

In section 3, we pointed out that SA can expand its market size by coordinating pricing strategy in the case where SA's pricing strategy does not affect the retail prices in SC. Section 4 shows that a discount ticket system serves a function to coordinate pricing strategy then each shop in SA can increase their profit by introducing this system.

Now we put some assumptions toward a discount ticket system in this paper. At first, a club-like organization (hereinafter called the club) is formed. Each shop in SA can choose whether they belong to the club or not, thus not all shops in SA join the club. Only if they join the club, they can issue a discount ticket. Comsumers can observe, before shopping, what shops belong the club.

We define the equilibrium price in a decentralized way model as the usual price. Consumers can get a discount of $2s$ compared with usual price if they purphase 2 kinds of comodities in the shops joining the club and issuing the discount ticket. s means discount and is determined to maximize the joint profit of the shops belonging to the club. Consumers, however, cannot receive a discount with 1 or 0 ticket. That is to say, Comsumers have to pay usual price for both commodities in the case where they purchase one commodity in shop belonging to the club and the other commodities in shop not belonging to the club.

In this section we suppose a situation that n shops out of N shops are belonging to the club. we denote the set of the shops that belong to the club by Ω_1 and the set of the shops that don't belong to the club by Ω_0 .

4.2 Model Formulation of Consumers

When consumers ramdomly choose goods $G_i,\ G_j$, we can consider following 4 cases about the combinations of (G_i, G_j). There exists a bifurcation point, as we have seen in the basic model, where comsumers' utilities are identical in terms of a shop choice.

1) $S_i \in \Omega_1,\ S_j \in \Omega_1$

$$r_{ij}^2(p_i^{s*}, p_j^{s*}, p_i^l, p_j^l, s) = \frac{(p_i^l + p_j^l) - (p_i^{s*} + p_j^{s*}) + 2s + 1}{2} \tag{40}$$

2) $S_i \in \Omega_1,\ S_j \in \Omega_0$

$$r_{ij}^{10}(p_i^{s*}, p_j^{s}, p_i^l, p_j^l) = \frac{(p_i^l + p_j^l) - (p_i^{s*} + p_j^{s}) + 1}{2} \tag{41}$$

3) $S_i \in \Omega_0,\ S_j \in \Omega_1$

$$r_{ij}^{01}(p_i^{s}, p_j^{s*}, p_i^l, p_j^l) = \frac{(p_i^l + p_j^l) - (p_i^{s} + p_j^{s*}) + 1}{2} \tag{42}$$

4) $S_i \in \Omega_0,\ S_j \in \Omega_0$

$$r_{ij}^{0}(p_i^{s}, p_j^{s}, p_i^l, p_j^l) = \frac{(p_i^l + p_j^l) - (p_i^{s} + p_j^{s}) + 1}{2} \tag{43}$$

For simplicity, we define $r_{ij}^2(p_i^{s*}, p_j^{s*}, p_i^l, p_j^l, s) = r_{ij}^2$, $r_{ij}^{10}(p_i^{s*}, p_j^{s}, p_i^l, p_j^l) = r_{ij}^{10}$, $r_{ij}^{01}(p_i^{s}, p_j^{s*}, p_i^l, p_j^l) = r_{ij}^{01}$, $r_{ij}^{0}(p_i^{s}, p_j^{s}, p_i^l, p_j^l) = r_{ij}^{0}$

4.3 Model Formation of Pricing Strategy

We formulate the expected profit function of shops that belong to the club and don't belong to the club. The expected profit function of $S_i \in \Omega_1$ can be expressed as

$$\pi_i^c = (p_i^{s*} - s - w_i) \sum_{\substack{j:S_j \in \Omega_1 \\ j \neq i}} r_{ij}^2 + (p_i^{s*} - w_i) \sum_{j:S_j \in \Omega_0} r_{ij}^{10} \tag{44}$$

The first term represents the profit that a shop can obtain from consumers who purchase the both commodities sold in the shop which join the club, and second term shows the profit from consumers who cannot receive a discount. Expected profit function of $S_i \in \Omega_0$ can be written as

$$\pi_i^e = (p_i^s - w_i)\left(\sum_{j:S_j\in\Omega_1} r_{ij}^{01} + \sum_{\substack{j:S_j\in\Omega_0\\ j\neq i}} r_{ij}^{0}\right) \quad (45)$$

The first term represents the profit from consumers who get one commodity in a club shop but the other commodity not in a club shop, and second term is the profit from consumers who get both commodities not in the club shops.

Behavior of shops in SA can be formulated as a problem where the club maximizes its joint profit with control variable s and shops not in the club determine their retail prices in a decentralized way, thus we can get the following equations.

$$\max_{s} \Pi^c \quad (46)$$

$$\max_{p_i^e} \pi_i^e \qquad (i \in \Omega_0) \quad (47)$$

where

$$\Pi^c = \sum_{i:i\in\Omega_1} \pi_i^c \quad (48)$$

The first order optimization condition of (43) be expressed as follows

$$\frac{\partial \Pi^c}{\partial s} = -\sum_{i:S_i\in\Omega_1}\sum_{\substack{j:S_j\in\Omega_1\\ j\neq i}} r_{ij}^2 + (n-1)\sum_{i:S_i\in\Omega_1}(p_i^{s*} - s - w_i) = 0 \quad (49)$$

The first term shows the decrease of profit caused by making a discount and the second term represents the increase of profit induced by the increase of the demand brought by a duscount. The second term depends on n which is the number of shops that belong to the club. Therefore, the more n is increasing, the more SA can internalize 'demand externality' as we will investigate in what follows. The first order condition of (44) can be written as

$$\frac{\partial \pi_i^e}{\partial p_i^e} = \left(\sum_{j:S_j\in\Omega_1} r_{ij}^{01} + \sum_{\substack{j:S_j\in\Omega_0\\ j\neq i}} r_{ij}^{0}\right) - \frac{N-1}{2}(p_i^s - w_i) = 0 \qquad (i \in \Omega_0) \quad (50)$$

In order to simplify following argument, we put some assumptions as below

1) Profit per unit of a commodity is identical to all shops in the club.
2) Profit per unit of a commodity is identical to all shops not in the club
3) Profit per unit of a commodity is identical to the SC.
4) N, the number of a commodities, is large enough.

Under these conditions, we can get Nash Equilibrium Solution in this model by solving (46) and (47) as simultaneous equation as follows;

$$s^{**} = \frac{2\hat{t}^l + 1}{12} \quad (51)$$

$$t^{s0**} = \frac{2\hat{t}^l}{3-\alpha} + \frac{1}{3} \quad (52)$$

Here, $\alpha = n/N$ and this represents the scale of the club. Bifurcation point r_{ij}^{**} can be expressed as

$$r_{ij}^2 = \frac{1}{2} + \frac{2\hat{t}^l - 1}{4} \quad (53)$$

$$r_{ij}^{10} = r_{ij}^{01} = \frac{1}{2} + \left(\frac{5-3\alpha}{2(3-\alpha)}\hat{t}^l - \frac{7}{12}\right) \quad (54)$$

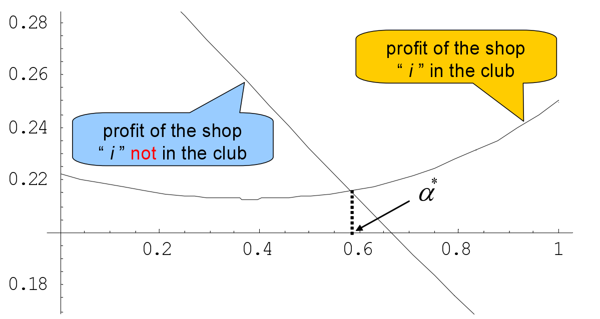
$$r_{ij}^{0} = \frac{1}{2} + \left(\frac{1-\alpha}{3-\alpha}\hat{t}^l - \frac{1}{3}\right) \quad (55)$$

Since all shops in the club enjoy the same profit π_i^{s1**} and all shops in the club get the same profit π_i^{s0**} under our assumptions, thus we can write such profits as follows

$$\pi_i^{s1**}(\hat{t}^l) = \alpha\left\{\frac{2\hat{t}^l+1}{4}\left(\frac{1}{2} + \frac{2\hat{t}^l - 1}{4}\right)\right\} + (1-\alpha)\left\{\frac{2\hat{t}^l+1}{3}\left(\frac{1}{2} + \frac{5-3\alpha}{2(3-\alpha)}\hat{t}^l - \frac{7}{12}\right)\right\} \quad (56)$$

$$\pi_i^{s0**}(\hat{t}^l) = \left(\frac{2\hat{t}^l}{3-\alpha} + \frac{1}{3}\right)\left\{\alpha\left(\frac{1}{2} + \frac{5-3\alpha}{2(3-\alpha)}\hat{t}^l - \frac{7}{12}\right) + (1-\alpha)\left(\frac{1}{2} + \frac{1-\alpha}{3-\alpha}\hat{t}^l - \frac{1}{3}\right)\right\} \quad (57)$$

Figure.2 shows that the expected function of shops in SA when $\hat{t}^l = 1/2$. Horizontal axis indicates α which is the scale of the club. As shown in **Figure. 2** when the size of the club is smaller than α^*, the shops in SA don't have incentive to join the club



horizontal axis means $\alpha = n/N$. In other words, scale of the club

Figure.2: Expected Profit of the Shops in SA

and the club will disappear as a rusult. When $\alpha > \alpha^*$, on the other hand, all shops not in the club have incentive to belong to the club. For all of these reasons, we have the following **Proposision 2**

proposition 2: By introducing the discount ticket system, the SA can expand its market size and increase the profit as a whole. However, to realize such a situation the scale of the club must exceed a fixed size α^*.

4.4 The Role of Discount Ticket System

In this model, we put some special assumptions, thus we menthion realistic suggestions for such assumptions. In our model, consumers can get a discount onlt when they collect 2 tickets. Actually we can find a situation where consumers can receive a discount service by collecting some discount points issued by the shops. Our assumptions reflect such a fact. More importantly, if comsumers purchase one commodities in the shops which don't belong to the club, they cannot receive a discount. Consider such a situation from shop's standpoint. If the club provide a discount not only to the consumers who buy commodities in the club but also to the customers who perchase commodities not in the club, the duscount may increase the demand of shops which don't belong to the club. This is the mechanism which induces 'demand externality'. However, if the club make a discount service to only the club shops, they can internalize the demand externality caused by a disount service.

5 Conclusion

In this paper, we pointed out the price of a commodity sold in one shop in the shopping area may affect the demand for a commodity which is sold in another shop located in the same area. We call such an externality as ' demand externality '. Shops in the shopping area fail to internalize the demand externality because they set prices of commodities in a decentralized way. We have theoretically pointed out that, due to such a demand externality, the shops in the shopping area enjoys relatively smaller market size than that of the shopping center who can determine the prices for multiple commodities in a centralized way. The equilibrium in this model results in inefficient market structure in terms of the total transportation costs in an economy. The paper also has shown that shops in the shopping area can coordinate their pricing strategies and expand their market size by issuing discount tickets to the customers, by which the respective shops in the shopping area can internalize the demand externalities. However in order to maintain the scale of the club, due to increasing return to scale, it is needed that a certain number of shops have to agree on joining the club. We forgo a mechamism which can commit shops in the SC to participate the club organization as future tasks.

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