

# Public Goods

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# Public Goods

- **Non-rivalness (非競合性)**

The use of a good by one individual does not limit the amount of the good available for consumption by others.

- **Non-excludability (排除不可能性)**

It is impossible to exclude any individuals from consuming a good, *even if they do not pay for its use.*

\* **Public goods** are an example of a particular kind of consumer **externality**.

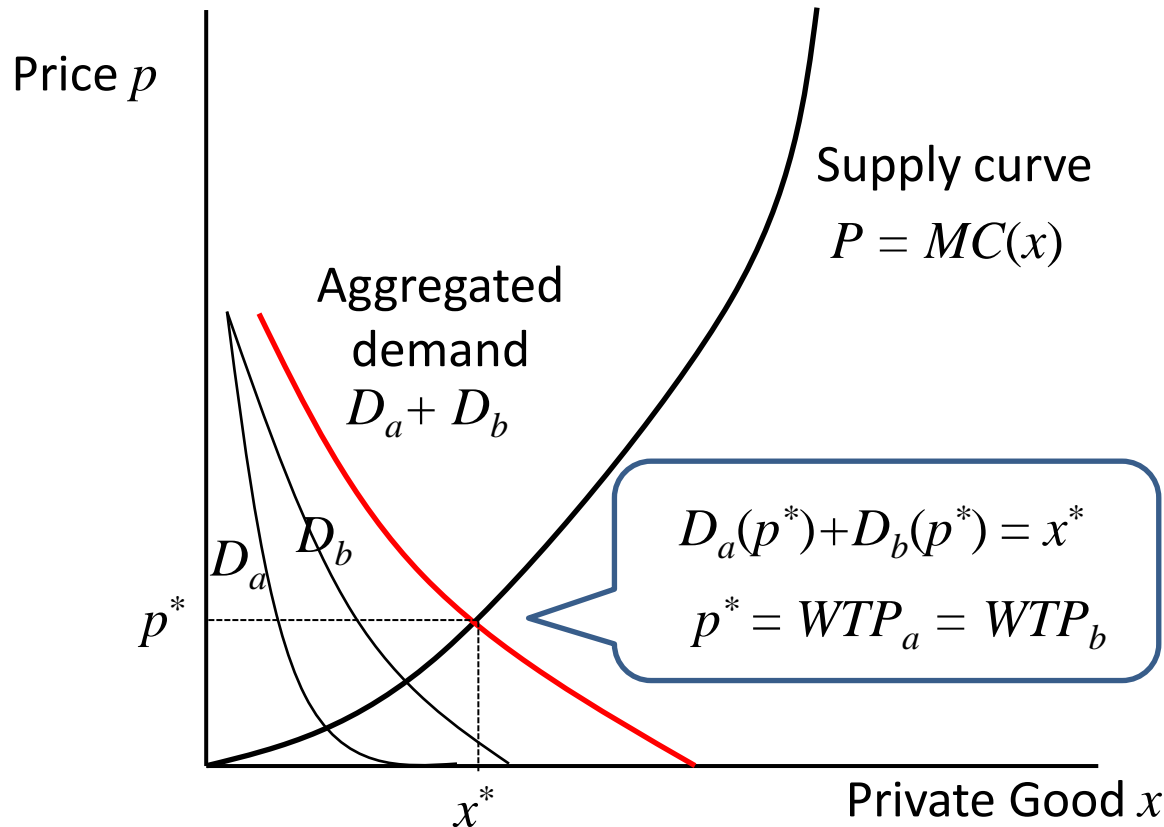
# Classification of goods

	Excludable	Non-excludable
Rivalrous	<b>Private goods</b> food, clothing, cars, computer, ...	<b>Common goods</b> fish stocks, timber, coal, ...
Non-rivalrous	<b>Club goods</b> expressway, private parks, knowledge, airport lounge, ...	<b>Public goods</b> national defence, embankment, ...

# Public goods and market failure

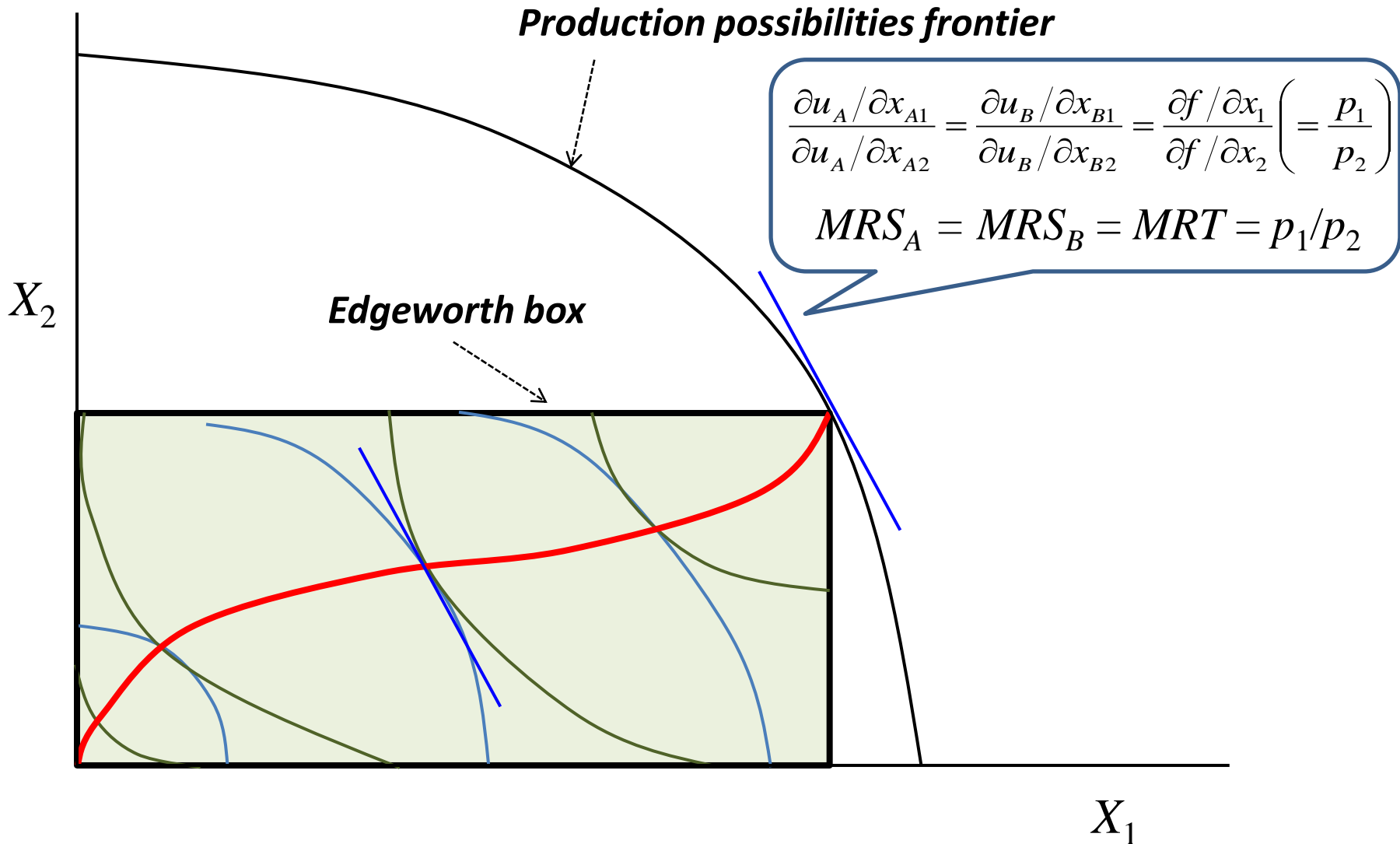
- **Non-rivalness (非競合性)**
  - **Under-supplied** (if the goods are supplied in market.)
  
- **Non-excludability (排除不可能性)**
  - **Free-rider problem**

# Marginal evaluation of **private good**

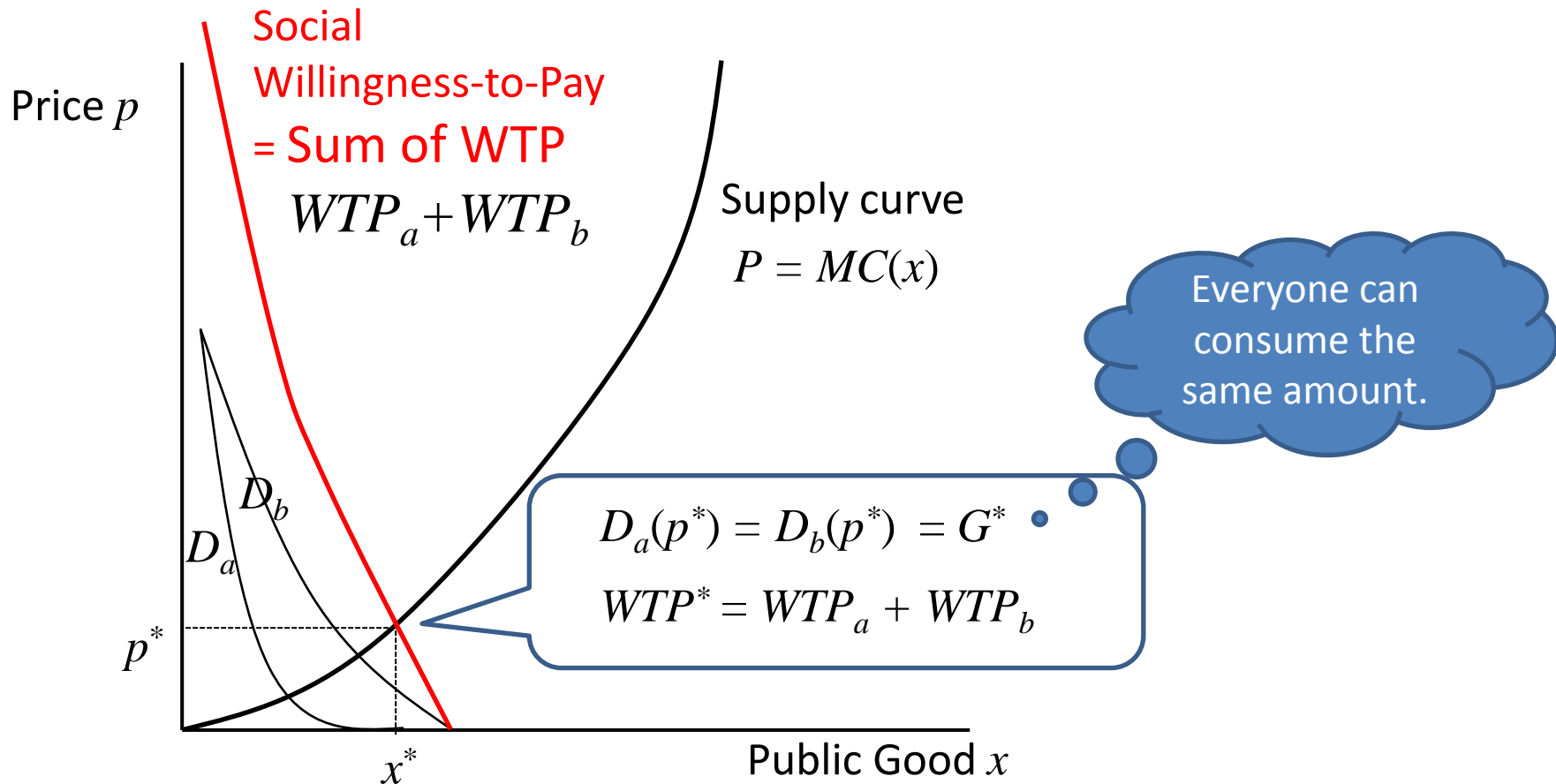


Price  $p$  = Willingness-to-pay for marginal consumption of the good

# Pareto efficient supply of private goods



# Marginal evaluation of **public good**



Price  $p$  = Willingness-to-pay for marginal consumption of the good

# Pareto-efficient supply of public goods

$$\max_{x_A, x_B, G} u_A(G, x_A)$$

Consumption of  
public goods

Consumption of  
private goods

subject to

$$u_B(G, x_B) \geq u_B^*$$

$$x_A + x_B + c(G) = w_A + w_B$$

Cost function  
of public  
goods

Initial allocation  
of wealth



# Pareto-efficient supply of public goods

- Lagrangian

$$L = u_A(G, x_A) + \lambda(u_B(G, x_B) - u_B^*) + \mu(w_A - w_B - x_A - x_B - c(G))$$

- First-order conditions

$$\left\{ \begin{array}{l} \frac{\partial L}{\partial x_A} = \frac{\partial u_A}{\partial x_A} - \mu = 0 \\ \frac{\partial L}{\partial x_B} = \lambda \frac{\partial u_B}{\partial x_B} - \mu = 0 \\ \frac{\partial L}{\partial G} = \frac{\partial u_A}{\partial G} + \lambda \frac{\partial u_B}{\partial G} - \mu \frac{\partial c(G)}{\partial G} = 0 \end{array} \right. \quad \Rightarrow \quad \frac{\frac{\partial u_A}{\partial G}}{\frac{\partial u_A}{\partial x_A}} + \frac{\frac{\partial u_B}{\partial G}}{\frac{\partial u_B}{\partial x_B}} = \frac{\partial c(G)}{\partial G}$$

$MRS_A$

$MRS_A$

$MC =$   
 $MRT$

# Pareto efficient supply of public goods

- **Samuelson condition** (サミュエルソン条件)

$$\sum_i MRS_i = MRT$$

Summation of  
*MRS* (marginal rate  
of substitution; 限界  
代替率)

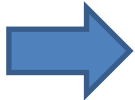
*MRT* (marginal rate  
of transformation; 限界  
変形率)

- In case of private goods,...

$$MRS_A = MRS_B = \dots = MRT$$

# If Household A **privately** provide the public goods...

- For simplicity,  $c(G) = G$


$$G(g_A + g_B) = g_A + g_B$$

- Utility maximisation problem of Household A

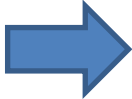
$$\max_{x_A, g_A} u_A(g_A + \bar{g}_B, x_A)$$

subject to

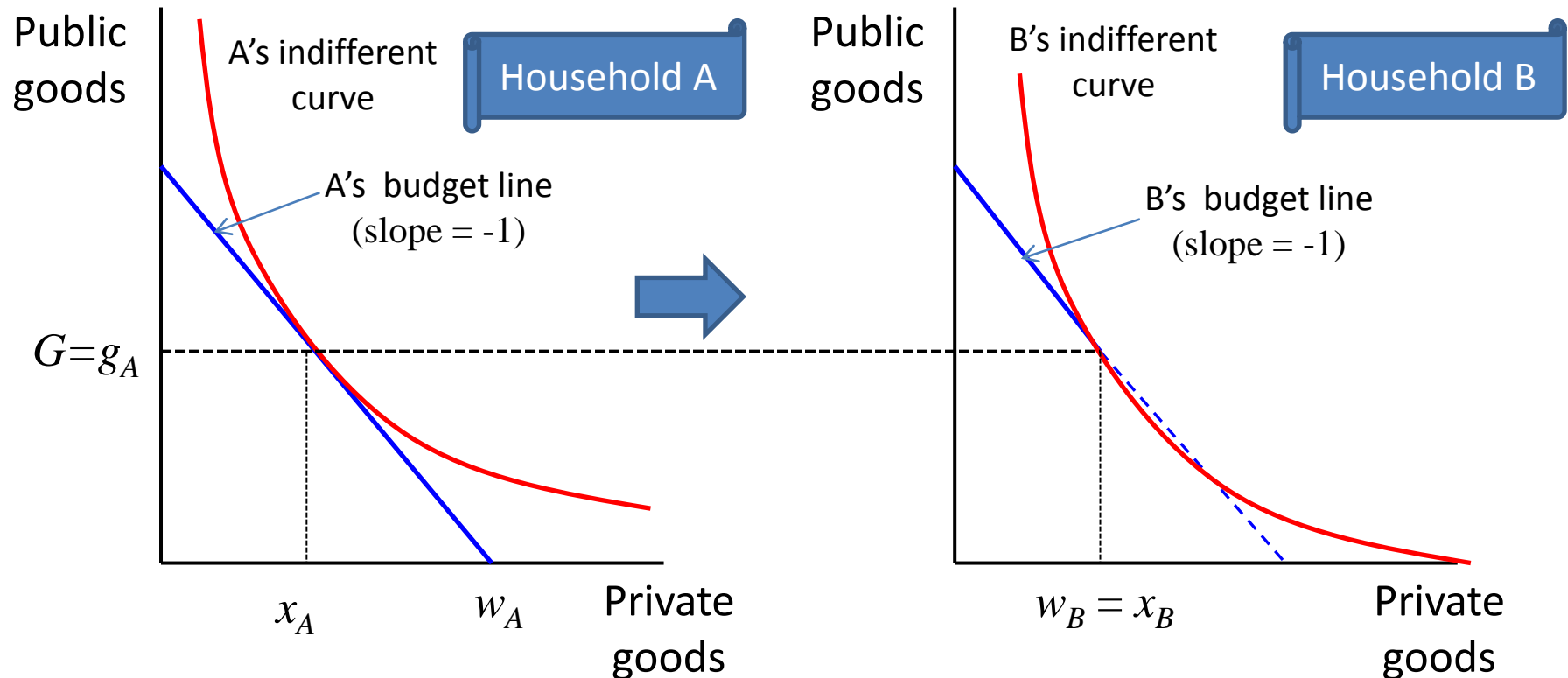
$$x_A + g_A = w_A$$

- First order condition

$$\frac{\partial u_A}{\partial G} \frac{\partial G}{\partial g_A} = \frac{\partial u_A}{\partial x_A}$$


$$\therefore MRS_A = \frac{\partial u_A}{\partial G} / \frac{\partial u_A}{\partial x_A} = 1$$

# Given that Household A purchases (produces) public goods...



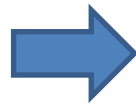
- Household B can consume public goods at  $G = g_A$  even if he does not pay at all.
- Therefore, the optimum strategy of Household B may be that he does not purchase public goods at all by himself.

# Free rider problem

A **free rider** is the one who enjoys benefits of goods **without paying** for them.

## Example

- Surface road
- Dam
- ...



Government's coordination of supply of public goods can be justified, if those public goods are under-supplied in market.