SPATIAL EQUILIBRIUM OF TAXI SPOT MARKETS AND SOCIAL WELFARE

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ABSTRACT

In this paper, how local behavior of agents affects global spatial equilibrium pattern is analyzed, in a matching model with taxies and passengers. There exist thick market externalities in taxi spot markets: the more taxies gather at a market, the more customers will visit the market, and vice versa. There always works the positive feedback mechanism in such ways that as more taxies and customers visit the market to transact the service, transaction costs can be reduced and the market functions more efficiently. In this paper, an equilibrium model of multi spot markets is presented considering both agglomeration mechanism caused by thick market externalities and dispersion mechanism caused by transaction costs. We have shown that there is a possibility of multiple equilibria, and analyzed the social efficiency of the respective equilibria.

Keywords: thick market externality, transaction cost, taxi market

1. INTRODUCTION

Transactions for taxi services occur in spot markets, such as at taxi bays settled in cities. Deregulation has accelerated price competition in setting taxi fees, but has thus far produced no notable differences between taxi companies, with taxies still offering customers rather homogeneous service. In spot markets, transactions for services are realized by matching the suppliers (taxis) with the consumers (passengers) of taxi services. The structure of taxi spot markets has heretofore not been studied. While macroscopic analysis containing about the deregulation of taxi markets, and theoretical and practical studies have appeared about taxi market equilibrium in a whole city, there have been few microscopic studies about spot markets. This paper proposes a market equilibrium model for analyzing the structure of a taxi spot market, in an attempt to remedy the dearth of studies explaining the self-organizing mechanisms of spot taxi markets.

In taxi markets, both the suppliers and the consumers are required to visit the spot market places to transact the services. Neither suppliers nor consumers are privy to full information about the state of supply and demand to be found at the spot, and both are required to decide whether they should visit the market place or not, based upon their imperfect guess about the current volume of market transactions. When suppliers and consumers of services are matched with each other at the market place, transactions for services are realized, while if neither appears in the market, no transaction takes place. If the agent finds no match at the spot, he must wait for the arrival of a match or leave the spot.

Both consumers and suppliers must cover transaction costs, which include travel costs to visit the market and the waiting costs necessary for finding a match at the spot market. Because of `imperfect guesses' and `transaction costs,' pecuniary externalities function to realize market transactions. For instance, if more suppliers visit the market and wait for the arrival of consumers, consumers can more easily find matching suppliers, and vice versa. More frequent visits to the spot market by both agents will give further payoffs to both. This phenomenon, called `thick-market externality,' means that the expectations of both suppliers and consumers that supply and demand will increase in fact bring about these increases. Likewise, expectations of lower supply and demand are also self-fulfilling. In a matching market with imperfect information and transaction costs, strategic complementarity caused by market interactions brings thick-market (thin-market) externality and causes multiplier effects in the market. In such markets with positive (negative) feedback effects, there is a possibility that multiple equilibria exist in the market.

On the other hand, in order to transact taxi service in a spot market, both passengers and taxies actually have to visit there. As travel cost to a market is key function for passengers to decide which market they use, the market with too much travel cost will not be chosen, even if it is the market which can serve a transaction with fewer waiting time. Consequently, all customers and taxies do not concentrate on one specific market, but service may be transacted in several markets. The spatial structure of spot markets in a certain area will be determined by correlation with the travel cost and the thick market externality. In a matching market with the mechanism of the centralization thorough thick market externality and decentralization with travel cost, there might exist multiple equilibria.

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There are several papers related to taxi markets. Among of all, Arnott presented a paper which showed the necessity of subsidy in urban taxi markets (Arnott 1996). Matsushima et al. formulated the model which explains endogenous market formation of taxi spot market. But these papers did not analyze the spatial equilibrium of taxi spot markets.

2. THE MODEL

(1) Assumptions

Let us take up the case where there are 2 spots market in a certain area and model the spatial equilibrium of spot markets from the behavior by customers and taxies. Assume a linear market with the length 2, which consider a business district along a street, as shown in Figure-1. Homogeneous visitors are uniformly distributed over the linear market. Spot markets are politically set at two places -*x* and *x*, by the traffic administrator with $0 \le x \le 1$. The only taxi spot market exists in the origin when *x*=0. Let us assume that *x*>0 is satisfied, and that the market with –*x* is indicated as *i*=1 and that with *x* as *i*=2. All customers can consume taxi services only at spot markets, and they do not have any other transportation service beside taxies. Passengers who arrive at spot markets will not leave until service is consumed, while there is maximum length for taxies queue with exogenous maximum length *M** (*M**=0,1,・・・,2). A taxi which arrive at the market when its queue length is *M** may leave from there. The mechanism which defines the maximum queue length of taxies will be expressed at **2. (3)**.

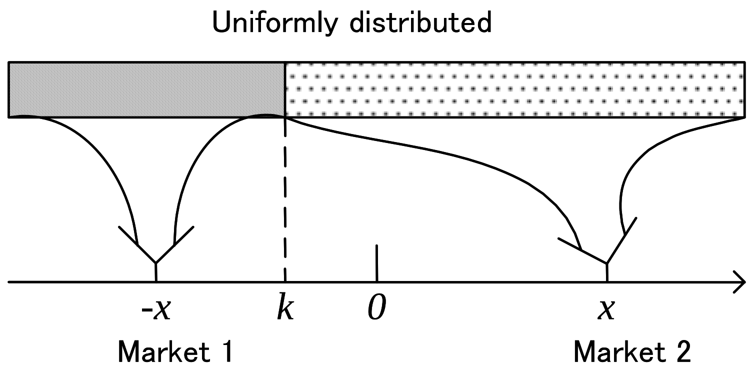

Figure 1 – Linear market

Passengers choose a spot market which maximizes their utility. Assume that the boundary of each market's territory exists at *k* ($-1 \le k \le k$). All passengers may use market 2 when *k*=-1, that shows market 1 disappeared, while only market 1 exists when *k*=1. Passengers arrive at both market subject to a Poisson process with arrival rate $\lambda_1 = (1+k)\zeta$ and $\lambda_2 = (1-k)\zeta$, respectively, where ζ shows exogenous average arrival rate of passengers per unit length and unit time at the linear market. On the other hand, average arrival rate of taxies will be ex pressed as $\mu_i\,(i=1,2)$. Though arrival rates of taxies are endogenously derived from market equilibrium, it is temporarily considered as exogenous (will be explained at 3.(1).

(2) Queuing model

Queuing phenomena, which are most typically observed in spot markets for taxi services, can be expressed by a double-queuing model first set forth by Kendall (1951) and further developed by Sasieni (1961). Though the double-queuing system proposed by Kendall is highly limited due to a failure to account for congestion, this system is adequate for investigating how spot taxi markets are autonomously organized via thick (thin)-market externalities.

Consider a consumer who arrives at a spot market. If a supplier has been waiting for a customer there, the consumer can immediately purchase service from the supplier. But if no supplier is there, he is forced to join the consumers' queue that has already been formed. Assume that once the consumer arrives at the market, he dose not leave until he makes a transaction with his supplier. In other words, there is no limit to the length of the consumers' queues. On the other hand, the length of the suppliers' queues is assumed to have a limit, due to the fact that suppliers are assumed to leave the market immediately if they find that the length of the suppliers' queue has already reached the upper bound *M*. The arrivals of both agents to the spot market are assumed to be purely stochastic and subject to a Poisson process with average arrival rates of the consumers λ and of the suppliers μ . For a moment, both λ and μ are assumed to be exogenous. This assumption will later be relaxed. It is also assumed that the coupled agents leave the market immediately after the service transaction has been completed between them.

Let us first consider a situation where only consumers are forming queues. Let the probability that the length of the consumers' queues is $n(n \geq 1)$ be $P(n)$. The transition mechanism of the system is described as

$$\begin{aligned} P_n(t+\Delta t) &= (1-\mu\Delta t)(1-\lambda\Delta t)P_n(t) + (1-\mu\Delta t)\lambda\Delta t P_{n-1}(t) \\ &\quad + \mu\Delta t(1-\lambda\Delta t)P_{n+1}(t) + o(\Delta t)! \end{aligned} \tag{1}$$

$o(\Delta t)!$ is the higher order term and $o(\Delta t)!/\Delta t \to 0$ as $\Delta t \to 0$. On the contrary, consider the opposite situation where only suppliers are forming queues. Denote the probability that *m* suppliers are waiting for the consumers' arrivals by $Q_m(t)$. The transition of the system is given by

$$Q_M(t+\Delta t) = (1-\lambda\Delta t)Q_M(t) + (1-\lambda\Delta t)\mu\Delta t P_{M-1}(t) + o(\Delta t)! \tag{2}$$

$$\begin{aligned} Q_m(t+\Delta t) &= (1-\mu\Delta t)(1-\lambda\Delta t)Q_m(t) + (1-\lambda\Delta t)\mu\Delta t Q_{m-1}(t) \\ &\quad + \lambda\Delta t(1-\mu\Delta t)Q_{m+1}(t) + o(\Delta t)!, \end{aligned} \tag{3}$$

where $o(\Delta t)!/\Delta t \to 0$ as $\Delta t \to 0$. Defining the probability with no queues by $P_0(t) = Q_0(t)$, the state equation is given by

$$\begin{aligned} Q_0(t+\Delta t) &= (1-\lambda\Delta t)(1-\mu\Delta t)Q_0(t) + \lambda\Delta t(1-\mu\Delta t)Q_1(t) \\ &\quad + (1-\lambda\Delta t)\mu\Delta t P_1(t) + o(\Delta t)!, \end{aligned} \tag{4}$$

Dividing both sides of eqs. (1), (2), (3), and (4) by Δt and considering the $\Delta t \to 0$ limit, in the long-run steady states, we see that

$$-(\mu+\lambda)P_n + \lambda P_{n-1} + \mu P_{n+1} = 0 \tag{5}$$

$$-\lambda Q_M + \mu Q_{M-1} = 0 \tag{6}$$

$$-(\lambda+\mu)Q_m+\mu Q_{m-1}+\lambda Q_{m+1}=0 \tag{7}$$

$$-(\lambda+\mu)Q_0+\lambda Q_1+\mu P_1=0 \tag{8}$$

$$(n=1,\cdots,\infty;m=1,\cdots M).$$

For the stability of the steady states, it should hold that $\mu>\lambda$. From eqs. (5)-(8), the stationary probabilities of P_n and Q_m are respectively given by

$$P_n=\rho^{M+n}Q_M \quad (n=1,\cdots,\infty) \tag{9}$$

$$Q_m=\rho^{M-m}Q_M \quad (m=0,\cdots,M), \tag{10}$$

where $\rho=\lambda/\mu$. From this definition, we see that

$$\sum_{n=1}^{\infty}P_n+\sum_{m=0}^{M}Q_m=1. \tag{11}$$

Substituting eqs (9) and (10) to (11), we see that

$$Q_M=1-\rho. \tag{12}$$

Accordingly, the stationary probabilities P_n and Q_m are given by

$$P_n=(1-\rho)\rho^{M+n} \quad (n\geq 1) \tag{13}$$

$$Q_m=(1-\rho)\rho^{M-m} \quad (M\geq m\geq 0). \tag{14}$$

Let $M(M=1,2,\cdots)$ be the maximum length of the suppliers' queues. Given the average arrival rates of the customers and the suppliers (λ,μ), the average lengths of the suppliers' queue and the consumers' queue are given by

$$E(n:\lambda,\mu,M)=\frac{\rho^{M+1}}{1-\rho} \tag{15}$$

$$E(m:\lambda,\mu,M)=M-\frac{\rho}{1-\rho}\left(1-\rho^M\right), \tag{16}$$

respectively. The average waiting time of the consumers and that of the suppliers, denoted by $T(\lambda,\mu,M)$ and $S(\lambda,\mu,M)$ respectively, with the arrival rates (λ,μ), are given by

$$T(\lambda,\mu,M)=E(n:\lambda,\mu,M)/\lambda \tag{17}$$

$$S(\lambda,\mu,M)=E(m:\lambda,\mu,M)/\mu \tag{18}$$

The probability ξ that a newly arrived supplier leaves the market without joining the suppliers' queue is defined by

$$\xi=Q_M=1-\rho. \tag{19}$$

If $M=0$, the suppliers immediately leave the market if they find that no consumers are waiting for the arrivals of the suppliers. Then, the average waiting times of the consumers and of the suppliers are given by

$$T(\lambda,\mu,0)=\frac{1}{\mu(1-\rho)} \tag{20}$$

$$S(\lambda,\mu,0)=0, \tag{21}$$

respectively.

(3) Maximum length of the queue

So far, the maximum length of the suppliers' queue is assumed to be exogenous. In what follows, these values are supposed to be endogenously determined through interactions between suppliers' and consumers' behavior in the market. Suppose that there is no physical limit on the length of the suppliers' queues. Each supplier who arrives at the spot market observes the current length of the suppliers' queue and decides to join the queue or to leave. The average waiting time of the *m*th suppliers in the suppliers' queues, denoted by $W(m)$, is given by

$$W(m) = \frac{m}{\lambda}. \tag{22}$$

Then, the expected profit of the *m*th supplier in the suppliers' queues, denoted by $\Pi(m)$, is defined by

$$\Pi(m) = q - \frac{m}{\lambda} - d , \tag{23}$$

where *q* is the expected profit per unit service transaction (measured in terms of time value). Though a supplier does not know precisely the actual revenue before he is matched with his customer, he can estimate the expected revenue through his past experiences. Suppose that the suppliers must pay transaction cost *c* for visiting the spot market. In order for the suppliers to have intentions to visit the market, the condition

$$q \geq d \tag{24}$$

should be satisfied. If it holds that $q < c$, no suppliers visit the market. In turn, the spot market disappears. Suppliers will join the queue as long as they can expect positive average profits from the spot market. The transaction cost for visiting the market has already been sunk by the time the supplier arrives at the spot market. In a competitive market, the maximum length of the suppliers' queues is determined in such a way that the maximum number of the suppliers waiting in the queues is a number that can guarantee nonnegative expected profits. From the non-negativity condition of the profits, $\Pi(m) \geq 0$, the maximum length of the suppliers' queues, $M(\lambda)$, is defined by

$$M(\lambda) = \left[(q-d)\lambda \right], \tag{25}$$

where the notation [·] means the maximum natural number that does not exceed $q\lambda$, and λ is the average arrival rate of the consumers. If the capacity of the spot market is physically limited, the maximum length of the suppliers' queues is conditional upon physical capacity, denoted by $M^*(W,\lambda)$. Then, the maximum length is given by

$$M^*(W,\lambda) = \min\{W, M(\lambda)\}, \tag{26}$$

where $M(\lambda)$ is the unconditional maximum length of the suppliers' queues (25) and *W* is the capacity of the market.

3. SPATIAL MARKET EQUILIBRIUM

So far, the arrival rates of the consumers and the suppliers λ_i, μ_i are assumed to be exogenous. In the long run, the arrival rates of both agents are endogenously determined through interactions between the suppliers and the consumers at the spot market. The consumer will visit the market as long as the expected utility he can derive from a transaction exceeds his reserved utility level. The consumer is assumed to remain at the market until his transaction is completed. On the other hand, the suppliers will remain at the market to wait for the arrival of his customer only if the length of the suppliers' queue is below its upper bound. Suppliers are expected to cover the transaction costs of visiting the market, and they will visit the market as long as they can anticipate gaining non-negative expected profit. As a result of the free entry of agents into the market, the average arrival rates both of consumers and suppliers are endogenously determined in the long run.

(1) Supplier (Taxis)

Let us temporarily assume that the consumers' arrival rate λ is exogenously given. When the suppliers arrive at the market with the arrival rate μ, the probability that the consumers make queues is given by ρ from eq. (19). Let us denote the physical market capacity in the spot market by $W(W=0,1,\cdots)$. If it holds that $W \geq M\lambda$, we see that $M^*(W,\lambda)=M(\lambda)$ from eq. (26). Given this fact, from now on let us focus exclusively upon the case where it holds that $M=W\leq M(\lambda)$. A supplier newly arrived at the market immediately leaves, if he finds the queuing length has already reached its upper bound. From eq. (19), the probability that the newly arrived supplier leaves the market is given by $\xi = 1-\rho$. Suppliers who have left the market can only gain a profit of *-c*. They can remain in the market with the probability $1-\xi=\rho$ and get the expected profit of

$$\Pi = q - S'(\lambda,\mu,W) - c, \tag{27}$$

where $S'(\lambda,\mu,W)=S(\lambda,\mu)/\rho$ is the conditional average waiting time of the suppliers when they can enter the market. Note that the profit is measured in terms of time value. From eq. (18), we see

$$S'(\lambda,\mu,W)=\frac{1}{\mu\rho}\left\{W-\frac{\rho}{1-\rho}\left(1-\rho^W\right)\right\}. \tag{28}$$

If no queuing is allowed for the suppliers $W=0$, it holds that

$$S'(\lambda,\mu,0)=0. \tag{29}$$

The suppliers visiting the market can gain an expected profit of $E(\Pi,W)$ $(W=0,1,2,\cdots)$:

$$E(\Pi,W)=\rho\left\{q-S'(\lambda,\mu,W)\right\}-d=\rho q-S(\lambda,\mu,W)-d. \tag{30}$$

The suppliers will visit the market as long as they can anticipate a non-negative profit. Provided the arrival rate of the consumers μ is exogenously fixed, the long-term arrival rate of the suppliers, denoted by μ^*, is determined to levels that satisfy

$$\frac{\lambda}{\mu^*}q - S\left(\lambda,\mu^*,W\right)-d=0. \tag{31}$$

(2) Consumers (Passengers)

Let us indicate the utility of passengers who locate at *y* ($-1\leq y\leq 1$) acquire from consuming taxi services with *y*, waiting time at market *i* with t_i, and travel cost to the market *i* with $c_i(y)$, respectively. Assume a linear utility function:

$$V_i(y) = v - t_i - c_i(y). \quad (32)$$

The utility function is measured in terms of time value. The first term corresponds to the idiosyncratic utility gained by transaction of services, which includes disutility from travel time, fare and travel cost. The utility is deterministic for each individual consumer, but a random variable for anybody else. $c_i(y)$ indicates the cost for passengers to move from *y* to market *i*, and is expressed as follows.

$$c_1(y) = \alpha|-x-y| \quad (33)$$

$$c_2(y) = \alpha|x-y| \quad (34)$$

$\alpha \geq 0$ indicates a parameter. The expected utility of the representative consumer who purchases services at the market is given by

$$EV_i(y) = v - T_i(\lambda_i, \mu_i, M_i) - c_i(y). \quad (35)$$

Market $i^*(y)$ which a passenger located at point *y* chooses is expressed as

$$i^*(y) = \arg\max\{EV_1(y), EV_2(y)\}. \quad (36)$$

Boundary of market share as shown in Figure-1 can be defined as $k \in [-x, x]$ which satisfies $EV_1(k)=EV_2(k)$. Passengers located at k can acquire same utility by using either market. Passengers' arrival rates λ_i to each market are

$$\lambda_1 = (1+k)\zeta \qquad \lambda_2 = (1-k)\zeta, \quad (37)$$

respectively. Taxies' arrival rate is μ_i which satisfies equation (31). Let us assume the following equations can be satisfied at the market boundary.

$$\left.\frac{dEV_1(y)}{dy}\right|_{y=k^*} < \left.\frac{dEV_2(y)}{dy}\right|_{y=k^*} \quad (38)$$

Then $EV_1(k^* - \varepsilon) > EV_2(k^* - \varepsilon)$ and $EV_1(k^* + \varepsilon) < EV_2(k^* + \varepsilon)$ are satisfied for small ε. Following equations are satisfied from the definition of $c_i(y)$ and market equilibrium with boundary *k* is stable.

$$EV_1(y) \leq EV_2(y) \quad y \in [-1, k) \quad (39)$$

$$EV_1(y) \geq EV_2(y) \quad y \in (k, 1] \quad (40)$$

When equation (38) is not satisfied on the other hand, it is not guaranteed that equations (39) and (40) are satisfied. This equilibrium is not stable. Other market equilibrium makes only one market appear. $\lambda_1 = 2\zeta$, $\lambda_2 = 0$, and $\mu_2 = 0$ when *k*=1, and μ_1 is calculated from equation (31). Following equation is satisfied at *k*=1.

$$EV_1(1) > EV_2(1) = -\infty \quad (41)$$

As $EV_1(y) > EV_2(y) = -\infty$ for any $y \in [-1,1]$, this equilibrium is also stable. Another equilibrium where all passengers use market 2 is also stable. From the mentioned above, equilibrium solutions can be expressed as follows.

$$\overline{EV}_1 = \overline{EV}_2 \quad (42)$$

$$\overline{EV}_i = EV_i(k^*) \ (i=1,2) \quad \lambda_1^* = (1+k^*)\zeta \quad \lambda_2^* = (1-k^*)\zeta$$

$$\rho_i q - S_i(\lambda_i^*, \mu_i^*, M_i) - d = 0 \quad \text{when } k \neq -1 \text{ and } k \neq 1 \quad (43)$$

$$\lambda_1^* = 0 \quad \mu_1^* = 0 \quad \lambda_2^* = 2\zeta \quad \overline{EV}_2 = EV_2(k)$$
$$\rho_2 q - S_2\left(\lambda_2^*, \mu_2^*, M_2\right) - d = 0 \quad \text{when } k = -1 \tag{44}$$
$$\lambda_2^* = 0 \quad \mu_2^* = 0 \quad \lambda_1^* = 2\zeta \quad \overline{EV}_1 = EV_1(k)$$
$$\rho_1 q - S_1\left(\lambda_1^*, \mu_1^*, M_1\right) - d = 0 \quad \text{when } k = 1 \tag{45}$$

Expected utility for a passenger located at *k* is expressed as $W(y) = \max\{EV_1(y), EV_2(y)\}$. Arrival rate of passengers may be the level with $W(y)$=0 in the long term equilibrium.

(3) Market externalities and travel cost

From eqs. (15) and (16), we see that the average length functions of the consumers' and the suppliers queues, denoted by $E(n_i : \lambda_i, \mu_i, M_i)$ and $E(m_i : \lambda_i, \mu_i, M_i)$, are homogeneous with respect to λ_i and μ_i, respectively. For arbitrary $\mu_i > \lambda_i \geq 0$ and $\theta > 0$, it holds that

$$E(n_i : \lambda_i, \mu_i, M_i) = E(n_i : \theta\lambda_i, \theta\mu_i, M_i) \tag{46}$$
$$E(m_i : \lambda_i, \mu_i, M_i) = E(m_i : \theta\lambda_i, \theta\mu_i, M_i), \tag{47}$$

which means that the average lengths of the consumers' and the suppliers' queues remain unchanged, even though the arrival rates of both agents simultaneously increase with the same rate θ. For arbitrary $\mu_i > \lambda_i \geq 0$ and $\theta > 1$, we see, from eqs (17) and (18), that

$$T_i(\lambda_i, \mu_i, M_i) = \theta T_i(\theta\lambda_i, \theta\mu_i, M_i) \tag{48}$$
$$S_i(\lambda_i, \mu_i, M_i) = \theta S_i(\theta\lambda_i, \theta\mu_i, M_i). \tag{49}$$

As the average arrival rates of both agents increase, the average waiting time of both agents will decrease. Thus, thick-market externality works in such a manner that as the waiting time of both agents becomes smaller, the number of both agents who enter the market becomes market. On the other hand, if either of two parameters, λ_i or μ_i is changed with the rate θ, the other parameter being unchanged, we see that

$$T_i(\lambda_i, \mu_i, M_i) < T_i(\theta\lambda_i, \theta\mu_i, M_i) \tag{50}$$
$$S_i(\lambda_i, \mu_i, M_i) < S_i(\theta\lambda_i, \theta\mu_i, M_i). \tag{51}$$

which means that as the arrival rate of either of the two agents increases, the average waiting time of the concerned agent increases. Congestion occurs if the arrival rate of either of the two agents increases, that of the other agent being unchanged.

From eqs. (48) and (49), it is apparent that the expected waiting time will decrease if more suppliers and consumers enter the spot market. There exists an increasing-return-to-scale externality in market transactions at the spot market. When both agents, the consumers and the suppliers, expect more counterparts to visit the market, this reduces the thinness of the field of matches and induces a further round of increased transactions. This will encourage high levels of transactions, which in equilibrium will result in high aggregate demand and supply, thereby fulfilling the original expectations. By the same token, the expectation of low transactions can also be self-fulfilling, because the associated prospect of thin markets and longer waiting costs will discourage the arrivals of both agents, there being strategic complementarity arising from market interactions, specifically from thick-market externality.

AS total number of passengers is limited, this positive feedback mechanism works until one market covers all passengers, as far as there is no power to control that. However, the existence of travel cost works as a key factor to control feedback mechanism. As number of markets decreases, number of passengers covered by one market increases to make passengers located around market border travel longer. These passengers may use closer market even if waiting time there is longer than that of a market far from her. The structure of spot market is endogenously determined thorough interaction between thick market externality and travel cost.

(4) Numerical examples

Let us analyze the impact of potential number of passengers on market equilibrium through numerical examples. Let us design the benchmark case (Case a) where the passengers' appearance rate ζ is 1.5, passengers expected utility v=10, taxies' utility profit q=10, taxies' transaction cost d=5, and a parameter of travel cost $\alpha = 1$. Set two spot markets at the edge of linear market (*x*=-1.0, *x*=1.0). The relation between the point of spot markets and social welfare will be analyzed in section 5. In order to make the discussion clear, queuing length of taxies are defined thorough market equilibrium, that is, we do not set any physical maximum length for taxies' queuing.

Let us assume that there exists boundary point y in the linear market, and that those who are located in the range *z* (*z*<*y*) visit market 1, while those who are located in the range *z'* (*z'* < *y*) visit market 2. Figure-2 shows the expected utility $EV_i(y)$ of passengers who visit either market 1 and 2. Passengers located at *y* may choose either market which makes their expected utility maximum. For those passengers who are located at the position where expected utility for both markets are the same ($EV_1(y^*)=EV_2(y^*)$), both markets are identical. Such a point y* shows the market boundary. Figure-2 shows 5 market boundaries from A to E. Each market boundary corresponds to one market equilibrium. Let us consider equilibrium with market boundary B. Assume that the market boundary moves to a point slightly right-hand side of B accidentally. As expected utilities of passengers located at *y*' are $EV_1(y')$ and $EV_2(y')$ with $EV_1(y') > EVU_2(y')$, they may use market 1. Point y' cannot be a market boundary, while point C will be. On the other hand, if market boundary move to left-hand side of B, $EV_1(y'') < EV_2(y'')$ is satisfied and equilibrium A will be new market boundary, that is, all passengers use market 2. Therefore, equilibrium B is unstable. Equilibrium D is also unstable in the same manner. There are 3 stable equilibrium, one with center market boundary, and other two where all passengers visit either market.

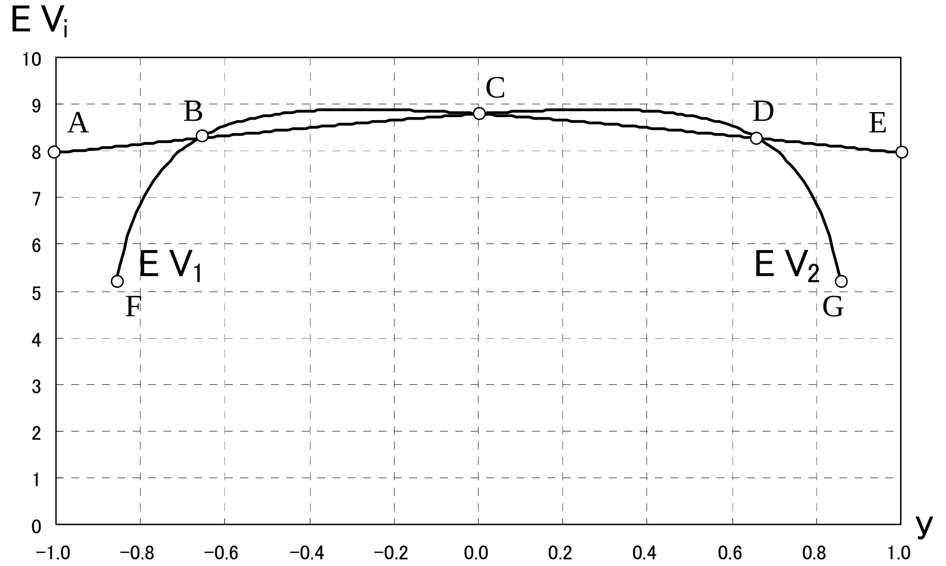

Figure-2 Market boundary and expected utility (Case a)

Figure-3 shows the same relation for a case with $\zeta = 0.5$ (Case b). There are 2 stable equilibria where all passengers visit both market and unstable equilibrium with central market border in this case. There is no equilibrium where both markets are visited. This means that thick market externality relatively conquer travel cost with low passengers density to make only one market survive.

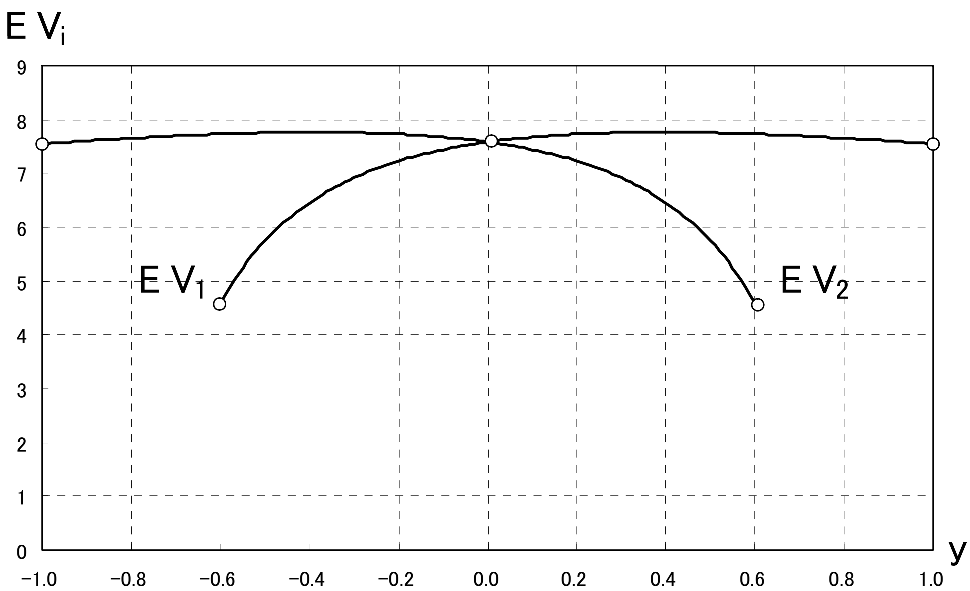

Figure-3 Market boundary and expected utility (Case b)

Figure-4 shows the relation between a parameter of travel cost α and density of passengers ζ. There are 2 areas in the figure: 1) 3 stable equilibria like case a, and 2) 2 stable equilibria like case b. There are possibly 2 equilibria as passengers' density ζ decreases, that is, all passengers visit either spot market. As travel cost α increases, equilibrium where both markets are visited like case a is available. Because the difference of travel cost increases as α increases, passengers tend to visit closer market.

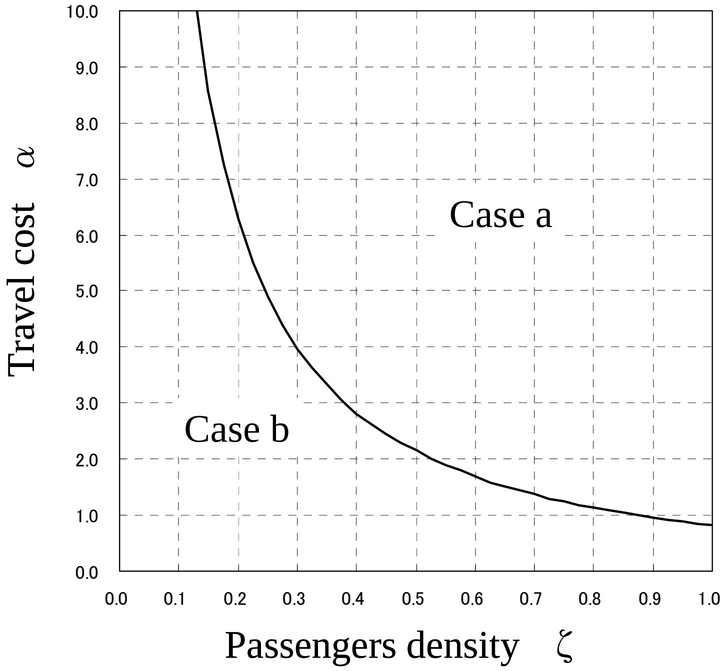

Figure-4 Passengers' density and market equilibrium

Numerical examples above shows that structure of market spatial equilibrium is different according to transaction cost and passengers' density. This means that multiple markets can co-exist at downtown areas with high dense of passengers, while in suburban are with low density it is difficult to have more than 1 spot market. Moreover, passengers and taxies visit the same market as travel cost increases to make another market disappear.

4. ARRANGEMENT OF SPOT MARKETS AND SOCIAL WELFARE

(1) Spatial pattern of spot markets

There possibly exist multiple spatial equilibria for taxi spot markets. 2 kinds of spatial equilibrium are possible; 1) 2 spot markets coexist, 2) only one market exits. Because thick market externality works as size of market becomes large, average waiting time for both taxies and passengers decreases. From the point of that view, it might be better that all passengers and taxies use only one market. However, that also brings about the increase of travel cost. Administrators should increase the number of market from the viewpoint of travel cost. Let us analyze spatial pattern of spot markets and social welfare in the following section.

(2) Generalization of market equilibrium model

Assume that there are *R* (*R*>0) spot markets on the linear market as shown in Figure-5. Let us indicate points of each market x_i (i=1, 2, ⋯, *R*) and area covered by market *i* $[k_{i-1}, k_i]$. $k_{i-1}=k_i$ means that market *i* is not realized, that is, nobody visits the market though it is set at a certain point. Taxies' behavior can be represented in the same way as 2 market model. Expected profit of taxies who visit spot market *i* is expressed as follows.

$$E(\Pi_i, M_i) = \rho_i q - S_i(\lambda_i, \mu_i, M_i) - d \tag{52}$$

As taxies enter market until when profit equals 0, arrival rate of taxies at market *i* (i=1, 2,⋯ , R) is μ_i which satisfies the following condition.

$$\rho_i q - S_i(\lambda_i, \mu_i, M_i) - d = 0 \tag{53}$$

Define passengers' utility at point *y* derived form visiting market *i* as equation (32). However, travel cost from point *y* to market *i* is written as follows.

$$c_i(x_i, y) = \alpha|x_i - y| \tag{54}$$

Expected utility of passenger *y* to visit market *i* is defined as follows.

$$EV_i(x_i, y) = v - T_i(\lambda_i, \mu_i, M_i) - c_i(x_i, y) \tag{55}$$

Generalizing equilibrium conditions (42)-(45), spatial equilibrium condition with R markets are equilibrium arrival rates (λ_i^*, μ_i^*) and market boundaries $k_i^* \ (i = 0, \cdots, R)$ which satisfy the following equations.

$$\overline{EV}^{+}_{i-1} = \overline{EV}^{-}_{i} \quad (i = 1, 2, \cdots, R) \qquad k_0 = -1 \le k_1 \le k_2 \le \cdots k_R = 1 \tag{56}$$

$$\overline{EV}^{-}_{i} = EV_i(x_i, k_{i-1}^*) \quad \overline{EV}^{+}_{i} = EV_i(x_i, k_i^*) \quad \lambda_i^* = (k_i^* - k_{i-1}^*)\zeta$$

$$\rho_i q - S_i(\lambda_i^*, \mu_i^*, M_i) - d = 0 \quad \text{when } k_i > k_{i-1} \ (i = 1, 2, \cdots, R) \tag{57}$$

$$\lambda_i^* = 0, \mu_i^* = 0 \quad \text{when } k_i = k_{i-1} (i = 1, 2, \cdots, R) \tag{58}$$

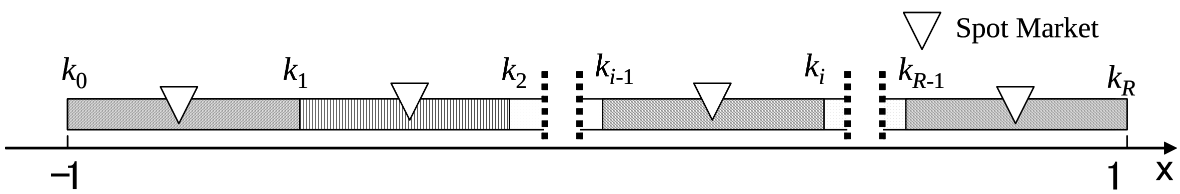

Figure-5 Location and boundaries for several markets

All markets in spatial equilibrium do not always exist. Assume that $k_{i-1}<k_i=\cdots k_{j-1}<k_j$. In this case, markets i+1, ⋯ ,j-1 are not realized. As $\lambda_s = 0, \mu_s = 0 \ (\text{s=i+1}, \cdots, \text{j-1})$, $EV_s(x_s, y) = -\infty \ (s = i+1, \cdots, j-1)$ for passengers at any position *y*. $EV_i(x_i, k_i) = \overline{EV}^{+}_{i} = \overline{EV}^{+}_{i+1} = \cdots = \overline{EV}^{-}_{j} = EV_j(x_j, k_{j-1})$ from equations (56)-(58). Boundary points $k_i=\cdots k_{j-1}$ indicate same position. Stability conditions of each equilibrium can be written as the same with former sections.

(3) Evaluation of social welfare

Transportation administrator can control social welfare with arrange spatial locations of spot markets. Consider a linear market with spot markets $\mathbf{x}_R = \{x_1, \cdots, x_R\}$. Social welfare *W*(*y*) for passenger $y \in [k_{i-1}, k_i]$ with equilibrium $\left(\lambda_i^*, \mu_i^*, (i=1,\cdots,R); k_i^*, (i=0,\cdots,R)\right)$ is defined as follows.

$$\begin{aligned} W(y) &= \max_i \{EV_i(x_i, y) : i = 1,\cdots,R\} \\ EV_i(x_i, y) &= v - T_i\left(\lambda_i^*, \mu_i^*, M_i\right) - c_i(x_i, y) \end{aligned} \tag{59}$$

Collective expected social welfare *CS* is expressed as follows.

$$CS(x) = \zeta \int_0^R W(y)\,dy \tag{60}$$

Expected profit of taxies is 0 in the long-term equilibrium, so producers' surplus is 0. Optimal spatial arrangement of spot markets is defined as combination of R and $\mathbf{x}_R$ which maximize equation (60) with constraints (56)-(58). If we consider more than 3 spot markets, there are too much spatial equilibrium which satisfies equations (56)-(58). It is very difficult to cover all possibilities. As this paper tries to analyze the mechanism of spatial equilibrium of spot markets, we stick to the cases with 2 markets.

(4) Numerical examples

Let us set two market with points *x* (0<*x*≤1) and –*x*. All parameters are set as the same with the case a in section 3. Though there possibly exists multi stable spatial equilibrium in this case, let us focus upon equilibrium where 2 spatial equilibrium coexists.Figure-6 shows the relation between market point *x* (0<*x*≤1) and social welfare. At *x*=0 in this figure, social welfare for the case with single spot market is also written. In both cases, spot market(s) should be located at the center of linear market(s). This case shows that social welfare with 2 markets is lager than that with single market.

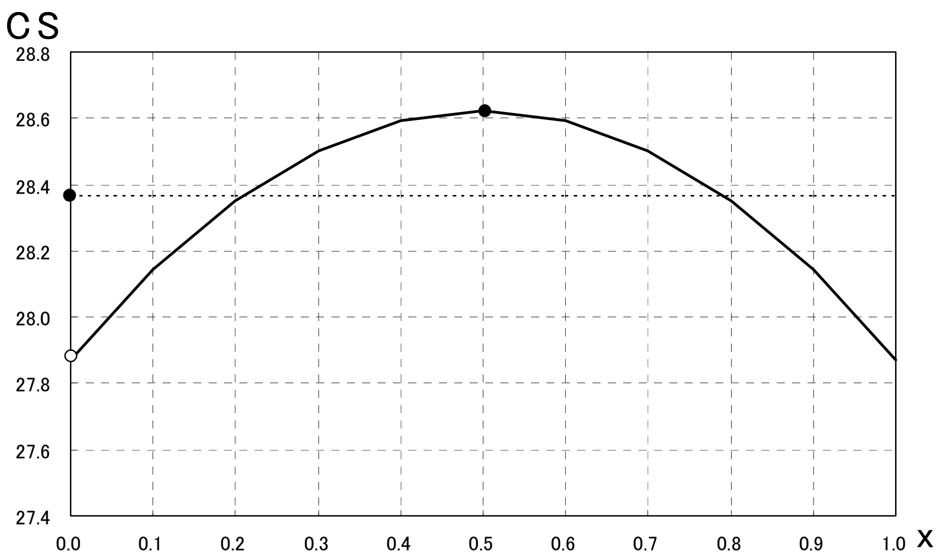

Figure-6 Market arraignment and social welfare ($\zeta = 1.5$)

Let us consider next the case with $\zeta = 1.0$. Though there are spatial equilibria with 2 markets, social welfare of that is smaller than that with single market (See Figure-7). This figure can explain the following things. Arrangement of 2 markets makes gross travel cost smaller than that with single market. However, average arrival rate at each market decreases as passengers and taxies are divided to 2 markets, which make average waiting time increase. When passengers' density is small enough in this case, disutility from the increase in the number of markets conquers utility from the decrease in gross travel cost. This result shows that social welfare is affected by passengers' density and travel cost to each market.

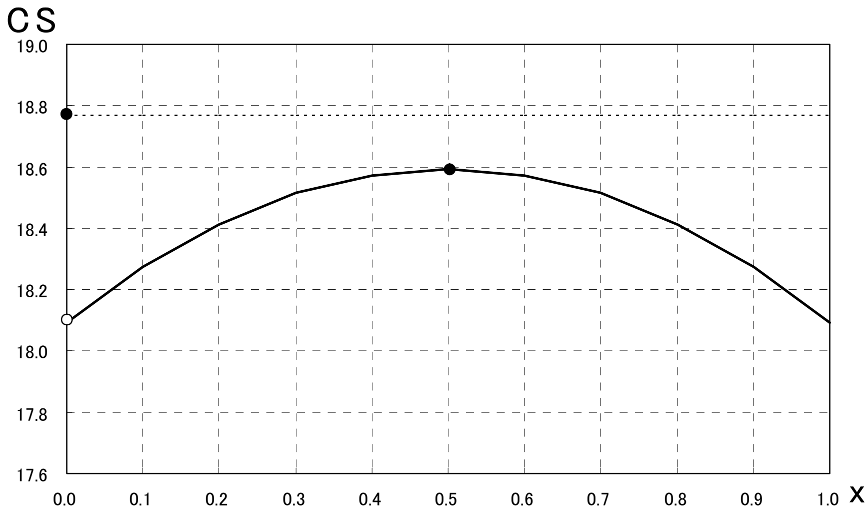

Figure-7 Market arraignment and social welfare ($\zeta = 1.0$)

Table-1 Passengers density and number of markets

Number of markets	$\zeta = 0.5$	$\zeta = 1.0$	$\zeta = 2.0$	$\zeta = 3.0$	$\zeta = 4.0$	$\zeta = 5.0$
1	9.05*	18.77	37.92	56.96	75.98	94.99
2	—	18.89*	38.54*	58.24	77.84	97.39
3	—	—	38.28	58.31*	78.19*	98.00
4	—	—	—	57.99	78.08	98.06*
5	—	—	—	—	77.33	97.86

Until now number of markets is set as 2 for simplicity. From now on, let us set spot markets as shown in Figure-5 in order to analyze the relation between the number of spot markets and social welfare. Table-1 shows the relation between number of markets and social welfare for several value of passengers' density. * indicates that the number of markets at this position maximizes social welfare for a certain passengers' density. – shows that this type of equilibrium cannot sustain no longer. Number of markets with stable spatial markets increases as passengers density ζ increases. Number of markets which maximizes social welfare also increases. Therefore, each market covers smaller range of market and passengers' arrival rate decreases to increase in expected waiting time, as the number of markets is increased. Travel cost to each market becomes smaller, on the other hand. As these 2 effects work to expected utility for passengers interactively, number of markets may affect social welfare complicatedly.

5. CONCLUSION

In taxi spot market where passengers and taxies are matched each other, thick market externality works, that is, service transaction becomes efficient thorough gathering many passengers and taxies. Because of scale economy related to this externality, more taxies and passengers use a certain market among several markets. On the other hand, passengers and taxies have to bear transaction cost in order to enjoy the service. If a certain market covers wider area, transaction cost of passengers located at the edge of linear market increases. Increase in transaction cost may decentralize markets. We had already analyzes the mechanism of single market formation in a city thorough thick market externality. This paper proposes the spatial equilibrium model where spatial equilibrium of spot market is formed through the mechanism of both co centralization and decentralization. Furthermore, the relation between spatial arrangement of spot markets and social welfare is analyzed.

This study should be extended in various directions. First, it is necessary to formulate optimal allocation model of spot market with this spatial equilibrium model. Second, the mechanism where passengers' density is endogenously determined should be analyzed. New equilibrium model which includes spatial equilibrium model should be formulated for the purpose. Third, information to both passengers and taxies should be considered. Giving proper information may make transaction more efficient.

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