ECONOMIC BENEFIT EVALUATION OF RESERVATION SYSTEMS

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Reservation system

- In this study, a reservation system is regarded as a mechanism, which allocates the services to potential customers along the first-come-first-served principle.

- Key wards
  [revelation mechanism, real option, economic benefits, monopolistic market]
Motivation

1. I want to know in this study what kind of influence the reservation system has on the consumers surplus, the company benefit, and the public welfare.

2. Necessity of the fare regulation.
The benefits of the reservation system

Option to purchase

- Reservation system supplies the customers an option to purchase certainly the service in the future.

Revelation mechanism

- Revelation mechanism attaches the priority to those who derive more satisfaction from the service than others along the first-come-first-served principle.
The preconditions of this model

- Monopolistic market
- A single service market.
- This model has 1\textsuperscript{st} and 2\textsuperscript{nd} term.
- There is limitation to a supply.
- There are two kinds of consumers.

Type H $\Leftrightarrow$ Type L
Type H ⇔ Type L

- Type H is the customers who get higher utility from the service than Type L and reserve at 1st term.

- Type L is the customers who get lower utility from the service than Type H and don’t reserve at 1st term.
The structure of consumer’s decision to reserve and purchase in this model

1st

Sub problem 1

reserve a service

not reserve a service

2nd

Sub problem 2

purchase a service

cancel the reservation

Sub problem 3

purchase a service

not purchase a service
The structure of Type H’s decision to reserve and purchase in this model

1st

Sub problem 1

reserve a service

not reserve a service

2nd

Sub problem 2

purchase a service

cancel the reservation

Sub problem 3

purchase a service

not purchase a service
The structure of Type L’s decision to reserve and purchase in this model

1st

- Sub problem 1
  - reserve a service
  - not reserve a service

2nd

- Sub problem 2
  - purchase a service
  - cancel the reservation

- Sub problem 3
  - Try to purchase a service
  - not purchase a service

- can
- cannot
The setting of this model

- The utility that Type H gets by the purchase of the service: \( u_H \)
- The utility that Type L gets by the purchase of the service: \( u_L \) \((< u_H)\)
- The number of Type H: 1
- The number of Type L: \( Q \)
The setting of this model

- The fare to pay when consumers purchase a service $p$
- The fare to pay when consumers canceled a service $c (\leq p)$
- The utility to get when consumers don’t purchase a service

\[ \mathcal{E} = \begin{cases} 0 & \text{in probability } q \\ \bar{\mathcal{E}} & \text{in probability } 1 - q \end{cases} \]
The behavior of Type H

1) If Type H reserved a service at 1\textsuperscript{st} term,

- The utility to get
  when Type H purchase a service at 2\textsuperscript{nd} term
  \[ V_H = u_H - p \]

- The utility to get
  when Type H canceled a service at 1\textsuperscript{st} term

\[ V_H = \varepsilon - c = \begin{cases} 0 - c & \text{in probability } q \\ \overline{\varepsilon} - c & \text{in probability } 1 - q \end{cases} \]
The behavior of Type H

If it is $0 - c < u_H - p < \bar{\epsilon} - c < \bar{\epsilon}$,

- The utility to get when Type H reserved at 1st

$$V_H = \begin{cases} 
    u_H - p & \text{When } \epsilon = 0 \text{ in probability } q \\
    \bar{\epsilon} - c & \text{When } \epsilon = \bar{\epsilon} \text{ in probability } 1-q 
\end{cases}$$

When we purchase the service

When we don’t purchase the service (cancel)
2) If Type H didn’t reserve a service at 1st,

- The utility to get
  
  when Type H purchase a service at 2nd

\[ V_H = u_H - p \]

- The utility to get
  
  when Type H don’t purchase a service at 2nd

\[ V_H = \mathcal{E} = \begin{cases} 
0 & \text{in probability } q \\
\bar{\mathcal{E}} & \text{in probability } 1 - q 
\end{cases} \]
The purchasability probability

- When consumers didn’t reserve at 1st term, there is those who cannot purchase at 2nd term. We can set purchasability probability in $h$.

1. Probability to try to purchase and get it: $q_h$
2. Probability to try to purchase but don’t get it: $q(1 - h)$
The behavior of Type H

\[ 0 - c < u_H - p < \varepsilon - c < \varepsilon \]

The utility to get when Type H didn’t reserve at 1st term

\[ V_H = \begin{cases} 
  u_H - p & \text{When consumers can get in probability } q_h \\
  0 & \text{When consumers cannot get in probability } q(1-h) \\
  \varepsilon & \text{When } \varepsilon = \bar{\varepsilon} \text{ in probability } 1-q 
\end{cases} \]
In summarizing of Type H’s behavior

If Type H did make a reservation

Type H get the utility

\[ V_H = \begin{cases} u_H - p & \text{in prob.. } q \\ \bar{\epsilon} - c & \text{in prob.. } 1 - q \end{cases} \]

If Type H didn’t make a reservation

Type H get the utility

\[ U_H = \begin{cases} u_H - p & \text{in prob.. } q h \\ 0 & \text{in prob.. } q (1 - h) \\ \bar{\epsilon} & \text{in prob.. } 1 - q \end{cases} \]
Expected utility of Type H

1) If we reserve a service at 1\textsuperscript{st} term, we get expected utility $EV_H$.

$$EV_H = q(u_H - p) + (1-q)(\bar{e} - c)$$

2) If we don’t reserve a service at 1\textsuperscript{st} term, we get expected utility $EU_H$.

$$EU_H = qh(u_H - p) + q(1-h)(0) + (1-q)\bar{e}$$

$$= qh(u_H - p) + (1-q)\bar{e}$$
3) Whether to reserve or not.

\[ EV_H \geq EU_H \implies \text{Type H reserve} \]

\[ EV_H < EU_H \implies \text{Type H don’t reserve} \]
WE can describe the expected utility of type L in the same manner of Type H.
Expected utility of Type L

1) If we reserve a service at t=0, we get expected utility $EV_L$.

$$EV_L = q(u_L - p) + (1 - q)(\bar{\epsilon} - c)$$

2) If we don’t reserve a service at t=0, we get expected utility $EU_L$.

$$EU_L = qh(u_L - p) + (1 - q)\bar{\epsilon}$$

3) Whether to reserve or not.

$$EV_L \geq EU_L \Rightarrow \text{Type L reserve}$$

$$EV_L < EU_L \Rightarrow \text{Type L don’t reserve}$$
Type H reserves and Type L doesn’t reserve. So, there is a possibility that just Type L cannot purchase the service. Type H purchases it in probability \( q \). The number of Type H is 1, so \( q \) people of them purchase it. Then just \( 1 - q \) services are left in the market for Type L. In other words, the supply for type L is \( 1 - q \) services. And the demand of Type L is \( qQ \). Therefore the purchasability probability is as follows,
The purchasability probability

\[ h = \frac{1 - q}{qQ} = \frac{\text{supply for Type } L}{\text{demand of Type } L} \]

The next condition is necessary to give the system a meaning.

\[ 1 - q < qQ \]

supply for Type \( L \) < demand of Type \( L \)
The constraints for revelation selection

\[ EV_H \geq EU_H \Rightarrow \text{Type H consumers reserve} \]

and

\[ EV_L < EU_L \Rightarrow \text{Type L consumers do not reserve} \]

\[
q(u_H - p) + (1-q)(\bar{E} - c) \geq qh(u_H - p) + (1-q)\bar{E} \\
q(u_L - p) + (1-q)(\bar{E} - c) < qh(u_L - p) + (1-q)\bar{E} \\
u_L \geq p
\]

More simply
The revelation selection constraints

\[
q(1-h)(u_H - p) \geq (1-q)c \\
q(1-h)(u_L - p) < (1-q)c \\
\text{and} \\
u_L \geq p
\]

\[\cdots \text{sc1} \]
\[\cdots \text{sc2} \]
\[\cdots \text{sc3} \]

The expectation of the amount of loss by canceling it when we made reservations

The expectation of the utility that we cannot get by not purchasing service when I did not make reservations
When the constraints are satisfied,

Type H reserve a service at $1^{st}$ term, 
Type L don’t reserve a service at $1^{st}$ term and
Type L try to purchase a service at $2^{nd}$ term.

Then,
the self-selection mechanism does work!
Profit maximization problem

\[ \text{max } p \]

Subject to

\[ q(1-h)(u_H - p) - (1-q)c \geq 0 \quad \dots \text{sc}1 \]

and

\[ q(1-h)(u_L - p) - (1-q)c < 0 \quad \dots \text{sc}2 \]

and

\[ u_L \geq p \quad \dots \text{sc}3 \]

The strongest limitation in these constraints on \( p \)

\[ p^* = u_L \]
\[ p^* = u_L \text{ satisfies } \]

\[
q(1-h)(u_H - p) - (1-q)c < 0 \quad \text{...sc2}
\]

\[
u_L \geq p \quad \text{...sc3}
\]
Profit maximization problem

\[
\max_c c \quad \text{Subject to} \quad q(1-h)(u_L - p) - (1-q)c \geq 0 \iff \frac{q(1-h)}{1-q}(u_L - p) \geq c
\]

\[
c^* = \frac{q(1-h)}{1-q}(u_H - u_L)
\]
The fares in reservation equilibrium

The fares in equilibrium

\[
\begin{align*}
p^* &= u_L \\
c^* &= \frac{q(1-h)}{1-q} \delta
\end{align*}
\]

A notice matter: \( \delta = u_H - u_L \)
The economic public welfare

**Total expected customer surplus of Type H** $EW_H$

$EW_H = Y + EV_H$

$= Y + q(u_H - p) + (1-q)(\bar{e} - c)$

**Total expected customer surplus of Type L** $EW_L$

$EW_L = Y + EU_L$

$= QY + Q\{qh(u_L - p) + q(1-h)(0) + (1-q)\bar{e}\}$

*The profit of the company* $\pi$

$\pi = p + (1-q)c - F$

*The total public surplus* $SW$

$SW = (1+Q)Y + q(u_H - p) + qQh(u_L - p) + (1-q)(1+Q)\bar{e} + p - F$
The fares in reservation equilibrium

The fares in equilibrium

\[
p^* = u_L \\
c^* = \frac{q(1-h)}{1-q} \Delta
\]

A notice matter: \( \Delta = u_H - u_L \)
The economic public welfare
in the reservation equilibrium

Total expected customer surplus of Type H $EW_H^*$

$$EW_H^* = Y + EV_H^*$$
$$= Y + qh(u_H - u_L) + (1 - q)\bar{\epsilon}$$

Total expected customer surplus of Type L $EW_L^*$

$$EW_L^* = Y + EU_L^*$$
$$= Y + (1 - q)Q\bar{\epsilon}$$

The profit of the company $\pi^*$

$$\pi^* = u_L + (1 - q)c - F$$

The total public surplus $SW^*$

$$SW^* = (1 + Q)Y + q(u_H - u_L) + (1 - q)(1 + Q)\bar{\epsilon} + p - F$$
\[ EW^*_H = Y + EV^*_H \]
\[ = Y + q(u_H - p) + (1 - q)(\bar{e} - c) \]
\[ = Y + q(u_H - p) + (1 - q)\bar{e} - (1 - q)c \]
\[ = Y + q(u_H - u_L) + (1 - q)\bar{e} - (1 - q)\frac{q(1-h)}{(1-q)}(u_H - u_L) \]
\[ = Y + q(u_H - u_L) + (1 - q)\bar{e} - q(u_H - u_L) + qh(u_H - u_L) \]
\[ = Y + qh(u_H - u_L) + (1 - q)\bar{e} \]
the standard equilibrium

*The purchasability probability in the standard equilibrium

\[ h^* = \frac{1}{q(1+Q)} = \frac{\text{supply for Type H and Type L}}{\text{demand of Type H and Type L}} \]

*The fare in the standard equilibrium

\[ p^* = u_L(= p^*) \]
The economic public welfare in the standard equilibrium

Total expected customer surplus of Type H $EW_H^*$

$$EW_H^* = Y + qh^*(u_H - u_L) + (1 - q)\bar{e}$$

Total expected customer surplus of Type L $EW_L^*$

$$EW_L^* = Y + EV_L$$

$$= Y + (1 - q)Q\bar{e}$$

The profit of the company $\pi^*$

$$\pi^* = u_L - F$$

The total public surplus $SW^*$

$$SW^* = (1 + Q)Y + qh^*(u_H - p) + (1 - q)(1 + Q)\bar{e} + u_L - F$$
### The economic effect of the reservation system

<table>
<thead>
<tr>
<th>The subject</th>
<th>Reservation equilibrium</th>
<th>Standard equilibrium</th>
<th>The effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type H</td>
<td>( Y + qh\delta + (1-q)\bar{e} )</td>
<td>( Y + qh^{\circ}\delta + (1-q)\bar{e} )</td>
<td>((h - h^{\circ})q\delta)</td>
</tr>
<tr>
<td>Type L company</td>
<td>( QY + (1-q)Q\bar{e} )</td>
<td>( QY + (1-q)Q\bar{e} )</td>
<td>0</td>
</tr>
<tr>
<td>Social surplus</td>
<td>( u_L + (1-q)c^* - F )</td>
<td>( u_L - F )</td>
<td>((1-q)c^*)</td>
</tr>
</tbody>
</table>

\[
\delta = u_H - u_L
\]
The analysis of the effect to Type H

The effect to Type H : $\Delta_H = (h - h^\circ)q\delta$

$\Delta_H = \left(1 - \frac{qQ - q}{Q(1 + Q)}\right)\delta < 0$ (Worse off)

Considering following limitation conditions

$h = \frac{1 - q}{qQ}, h^\circ = \frac{1}{q(1 + Q)}, 1 - q < qQ$
The analysis of the effect to the other objects

The effect to Type L: \( \Delta_L = 0 \)

The effect to company: \( \Delta_\pi = (1 - q)c^* \)

\( \Delta_L = 0 \)  Better off  \( \Delta_\pi \geq 0 \)
The analysis of the effect to social surplus

The effect to social surplus:

$$\Delta_{SW} = (1 - h^\circ)q\delta$$

$$\Delta_{SW} = \left(\frac{qQ + q - 1}{1 + Q}\right)\delta > 0$$

Better off

Considering following limitation conditions

$$h^\circ = \frac{1}{q(1+Q)}, 1 - q < qQ$$
The economic effect of the reservation system

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<td>Type H</td>
<td>((h - h^<em>)q\delta = (1 - h^</em>)q\delta - (1 - q)c^* &lt; 0)</td>
</tr>
<tr>
<td>Type L</td>
<td>0</td>
</tr>
<tr>
<td>company</td>
<td>((1 - q)c^* \geq 0)</td>
</tr>
<tr>
<td>Social surplus</td>
<td>((1 - h^*)q\delta &gt; 0)</td>
</tr>
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</table>

From this list, we can find that cancel fare moves the income from Type H to the company.