# Public Goods

# Public Goods

- **Non-rivalness (非競合性)**

  The use of a good by one individual does not limit the amount of the good available for consumption by others.

- **Non-excludability (排除不可能性)**

  It is impossible to exclude any individuals from consuming a good, *even if they do not pay for its use*.

* **Public goods** are an example of a particular kind of consumer **externality**.

# Classification of goods

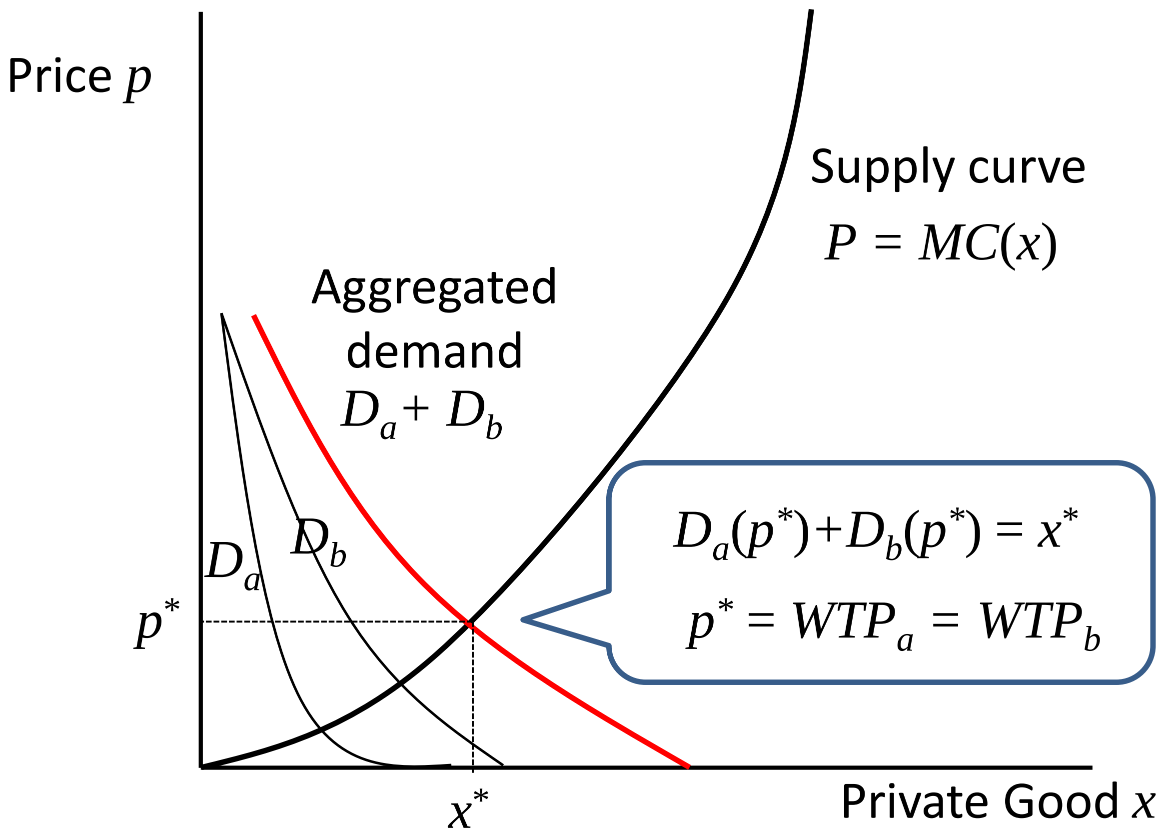
| | Excludable | Non-excludable |
|---|---|---|
| Rivalrous | **Private goods**<br>food, clothing, cars, computer, … | **Common goods**<br>fish stocks, timer, coal, … |
| Non-rivalrous | **Club goods**<br>expressway, private parks, knowledge, airport lounge, … | **Public goods**<br>national defence, embankment, … |

# Public goods and market failure

- **Non-rivalness (非競合性)**
  - **Under-supplied** (if the goods are supplied in market.)

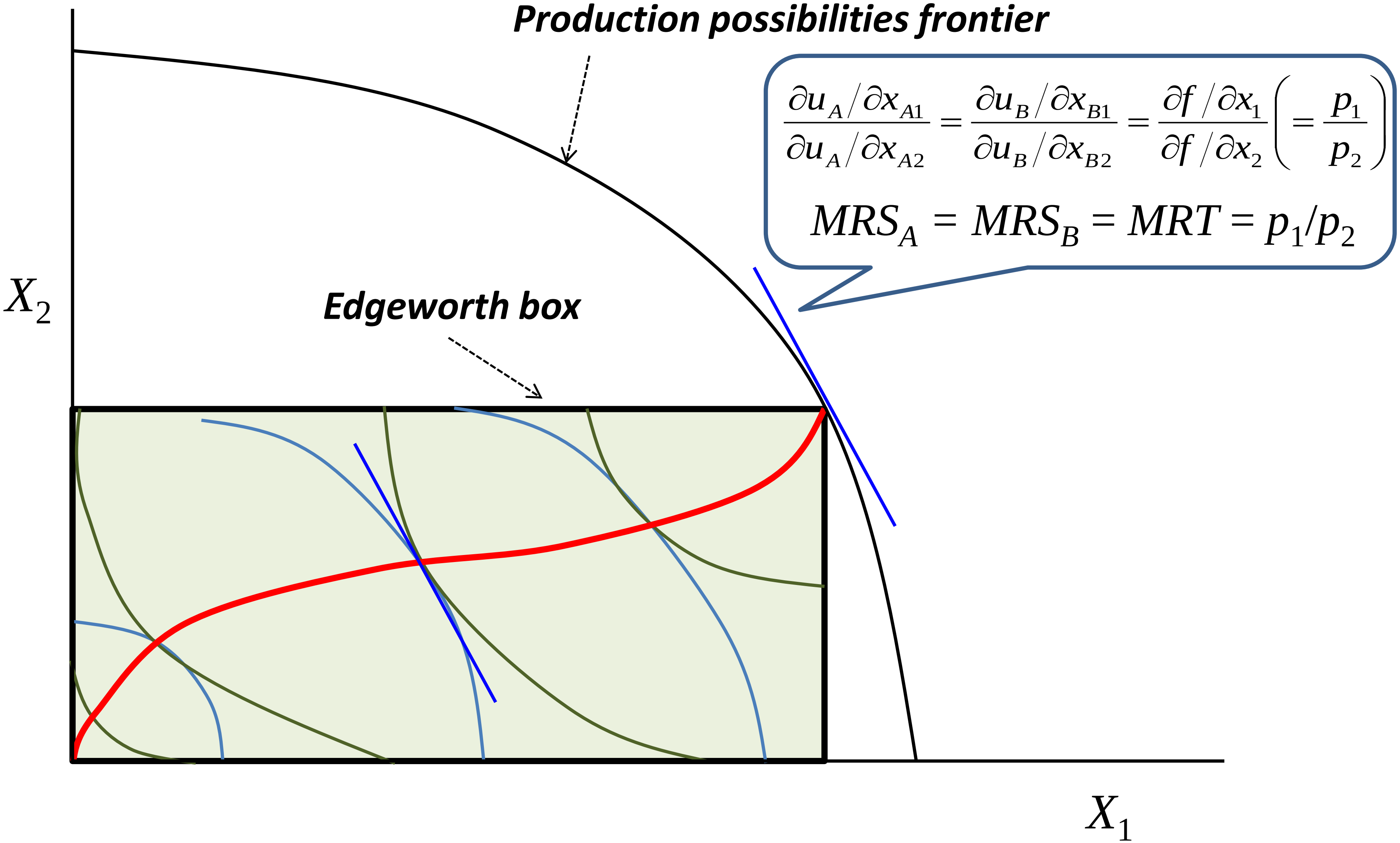
- **Non-excludability (排除不可能性)**
  - **Free-rider problem**

# Marginal evaluation of **private good**

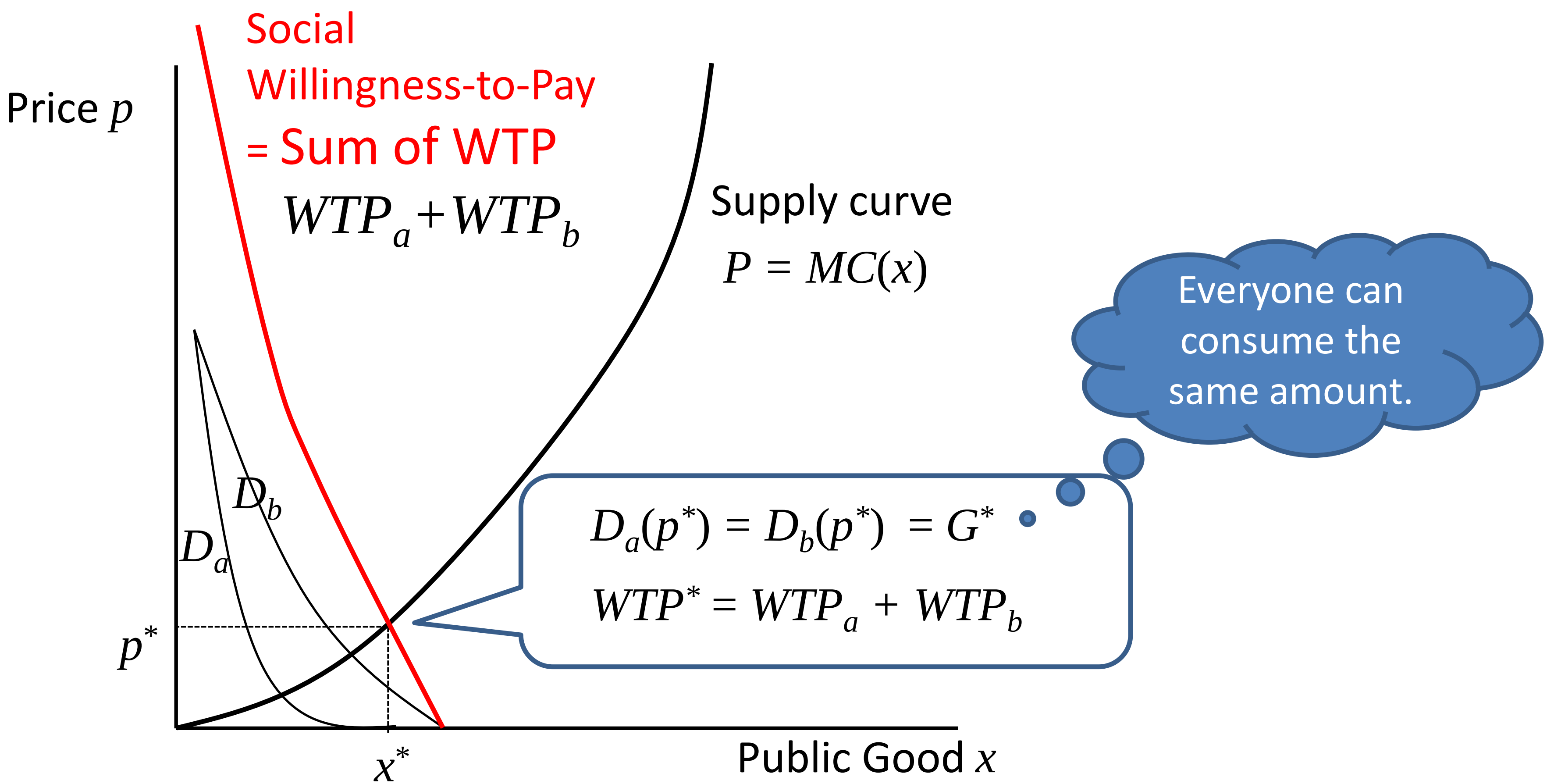



Price p = Willingness-to-pay for marginal consumption of the good

# Pareto efficient supply of private goods



# Marginal evaluation of **public good**




Price p = Willingness-to-pay for marginal consumption of the good

# Pareto-efficient supply of public goods

$$\max_{x_A, x_B, G} u_A(G, x_A)$$

Consumption of public goods

Consumption of private goods

subject to

$$u_B(G, x_B) \geq u_B^*$$

$$x_A + x_B + c(G) = w_A + w_B$$

Cost function of public goods

Initial allocation of wealth

# Pareto-efficient supply of public goods

- Lagrangian

$$L = u_A(G, x_A) + \lambda\left(u_B(G, x_B) - u_B^*\right) + \mu\left(w_A - w_B - x_A - x_B - c(G)\right)$$

- First-order conditions

$$\begin{cases} \dfrac{\partial L}{\partial x_A} = \dfrac{\partial u_A}{\partial x_A} - \mu = 0 \\ \dfrac{\partial L}{\partial x_B} = \lambda \dfrac{\partial u_B}{\partial x_B} - \mu = 0 \\ \dfrac{\partial L}{\partial G} = \dfrac{\partial u_A}{\partial G} + \lambda \dfrac{\partial u_B}{\partial G} - \mu \dfrac{\partial c(G)}{\partial G} = 0 \end{cases}$$

$$\Rightarrow \quad \frac{\partial u_A}{\partial G} \Big/ \frac{\partial u_A}{\partial x_A} + \frac{\partial u_B}{\partial G} \Big/ \frac{\partial u_B}{\partial x_B} = \frac{\partial c(G)}{\partial G}$$

$MRS_A$ | $MRS_A$ | $MC = MRT$

# Pareto efficient supply of public goods

- **Samuelson condition (サミュエルソン条件)**

$$\sum_i MRS_i = MRT$$

Summation of *MRS* (marginal rate of substitution; 限界代替率 )

MRT (marginal rate of transformation; 限界変形率)

- In case of private goods,...

$$MRS_A = MRS_B = \ldots = MRT$$

# If Household A **privately** provide the public goods…

- For simplicity, $c(G) = G$

$$\Rightarrow \quad G(g_A + g_B) = g_A + g_B$$

- Utility maximisation problem of Household A

$$\max_{x_A, g_A} u_A\left(g_A + \overline{g_B}, x_A\right)$$

subject to

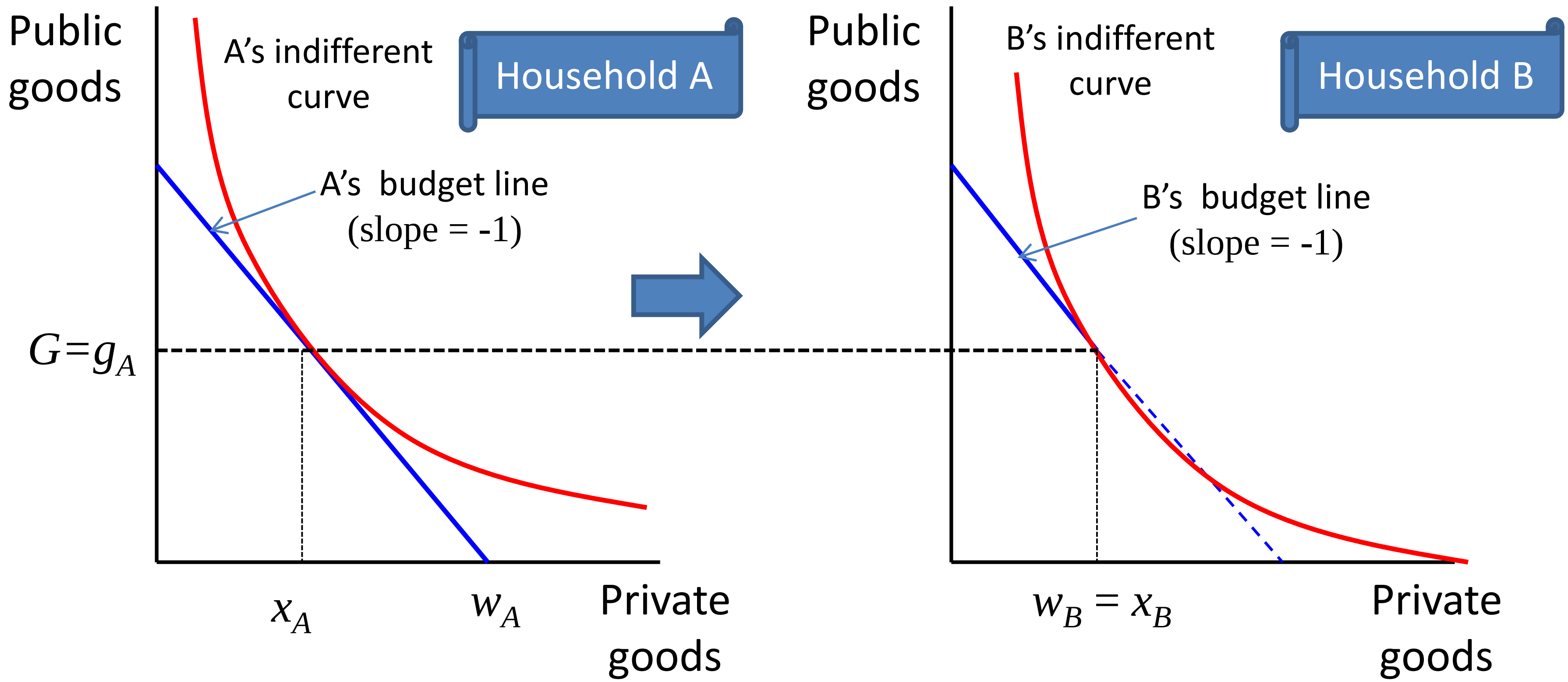
$$x_A + g_A = w_A$$

- First order condition

$$\frac{\partial u_A}{\partial G}\frac{\partial G}{\partial g_A} = \frac{\partial u_A}{\partial x_A}$$

$$\Rightarrow \quad \therefore MRS_A = \frac{\partial u_A}{\partial G} \Big/ \frac{\partial u_A}{\partial x_A} = 1$$

# Given that Household A purchases (produces) public goods...




- Household B can consume public goods at $G = g_A$ even if he does not pay at all.
- Therefore, the optimum strategy of Household B may be that he does not purchase public goods at all by himself.
  - Free-rider problem