Theory of Firm

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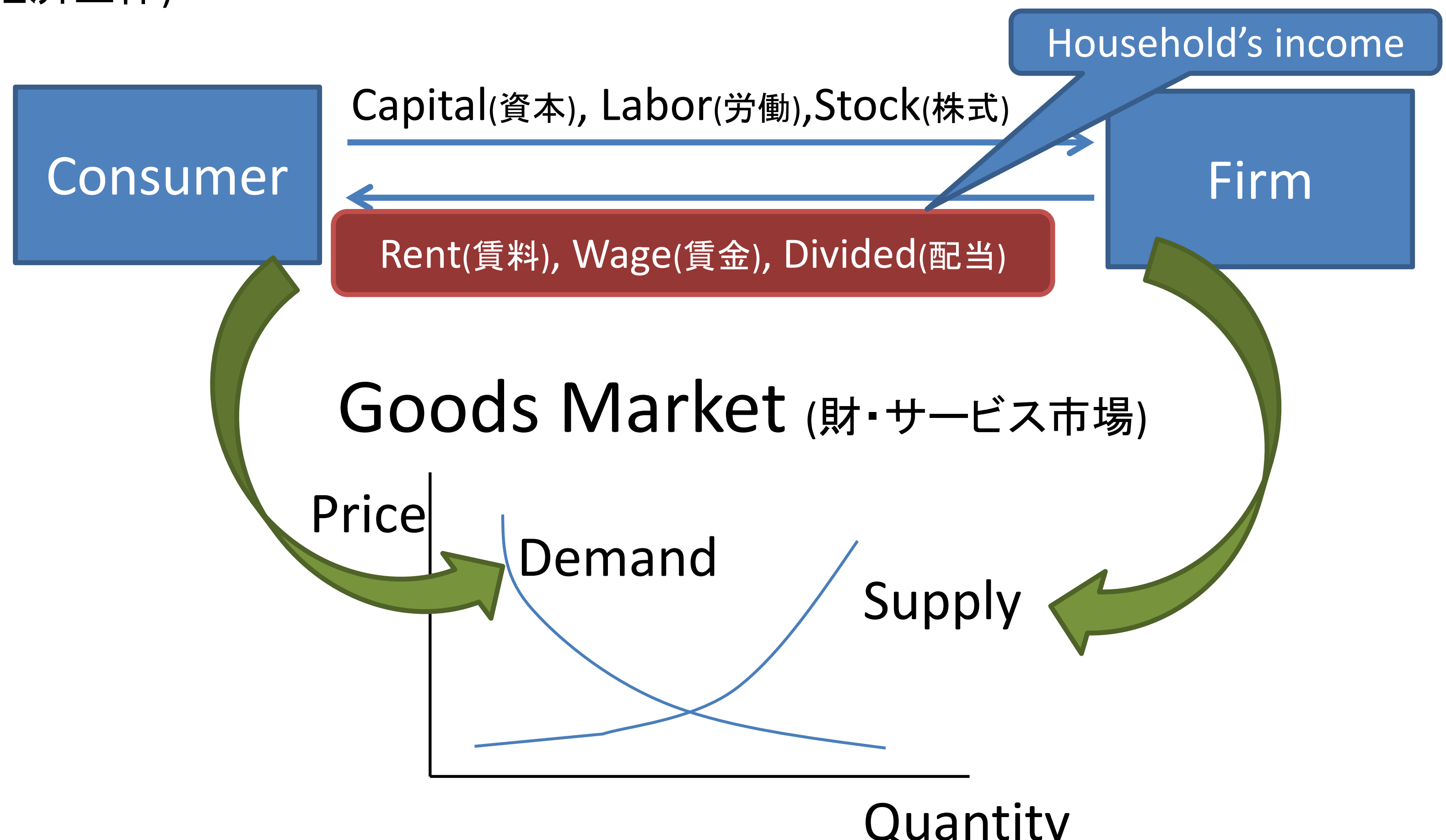
- Feature of Firm’s behaviour
- 企業の行動の特徴
- Cost minimization and Profit maximization
- 費用最小化と利潤最大化
- Cost function and Profit function
- 費用関数と利潤関数
- Market Supply Curve
- 市場供給関数
- Long run Equilibrium
- 長期均衡

Feature of Consumer Behaviour

Economic Entity
(経済主体)

Firm(企業), Consumer (家計), Government



Firm = Aim to maximising profit but not always be price taker

Profit

- Profit = Revenue – Cost
  - Revenue = Price X Quantity of output

    p y

  - Cost = Σ(Price X Quantity) of inputs (factors)

    w_i x_i

Constraints on Firm's behaviour

- Technological Constraints (技術的制約)
- Market Constraints (市場の制約)
  - Price mechanism that firm faces on

Market for outputs (産出物の市場)

Multiple player ➔ Price taker

Single supplier ➔ Monopoly (独占)

Market for the factors of production (生産要素市場)

Multiple recipient ➔ Price taker

Single recipient ➔ Monopoly (独占)

Competitive Market

Description of Technology (1)

- Technology is a system that transform input factors (生産要素) into production outputs (生産物)

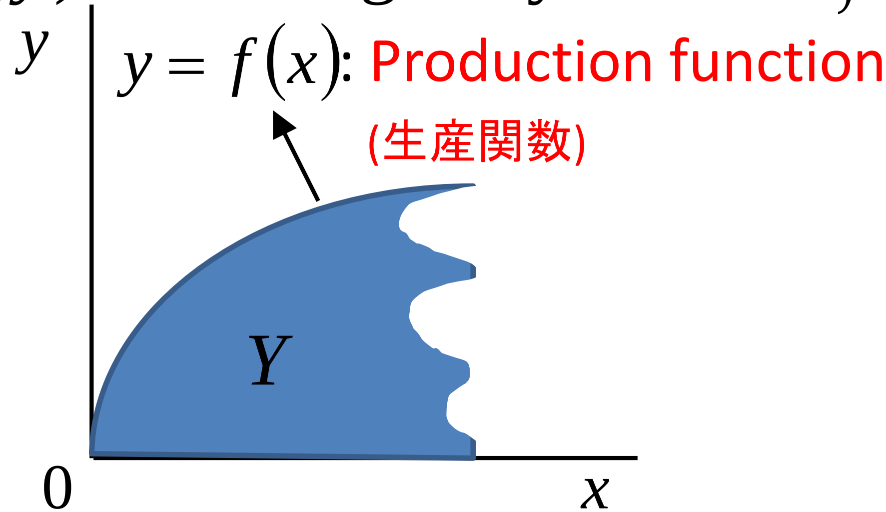
Production Possibilities Set (生産可能集合)

$$Y = \{(x,y), x \in R^n_+, y \in R^m_+ \mid (x,y) \text{ is technogically feasible.}\}$$

where

x: Amount of factors of production (input)

y: Amount of productions (output)

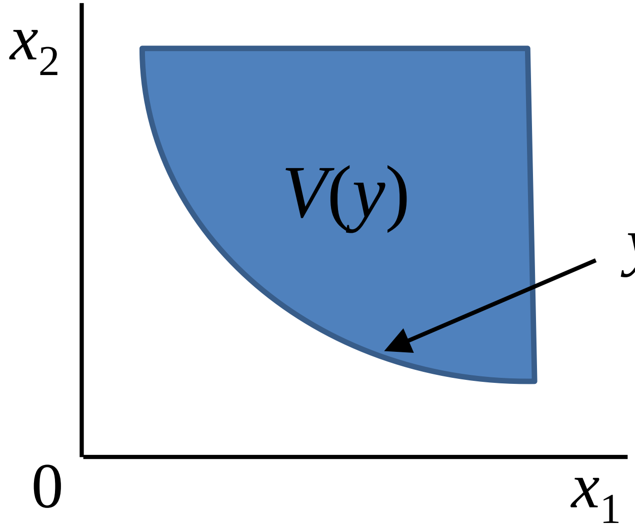


Description of Technology (2)

Input Requirement set (必要投入量集合)

$$V(y) = \left\{ x \in R_{+}^{n} \middle| (x,y) \in Y \right\}$$

Input requirement set is defined as that set of inputs required to produce at least a given amount of outputs, *y*



$y = f(x)$: Production function (生産関数)

Example of Production Function

- Leontief type

$$f(x) = \min\{a_1 x_1, \ldots, a_n x_n\}$$

- Cobb-Douglas type

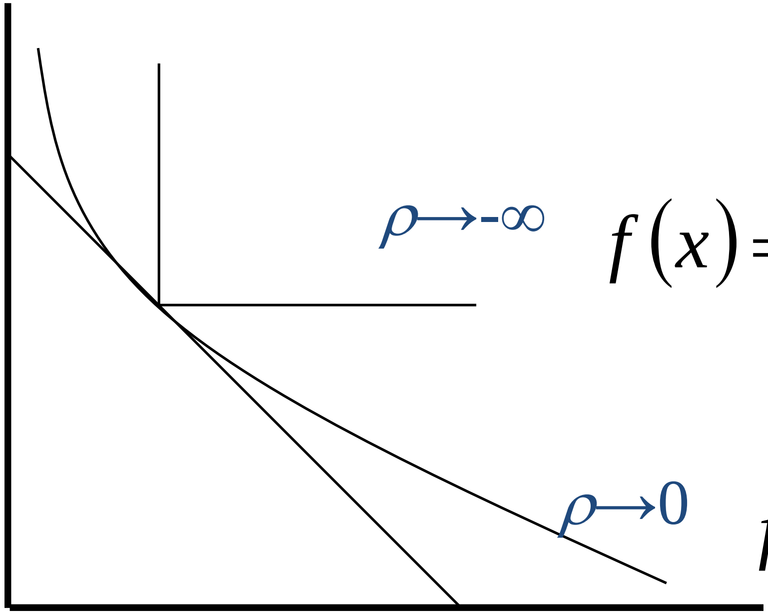
$$f(x) = a_0 x_1^{a_1} \cdots x_n^{a_n}$$

- Linear type

$$f(x) = a_1 x_1 + \cdots + a_n x_n$$

Example: CES type production function

$$f(x) = \left(a_0 + a_1 x_1^{\rho} + \cdots a_n x_n^{\rho}\right)^{1/\rho}$$



$\rho \to -\infty$ $$f(x) = \min\{a_1 x_1, \ldots, a_n x_n\}$$

$\rho \to 0$ $$f(x) = a_0 x_1^{a_1} \cdots x_n^{a_n}$$

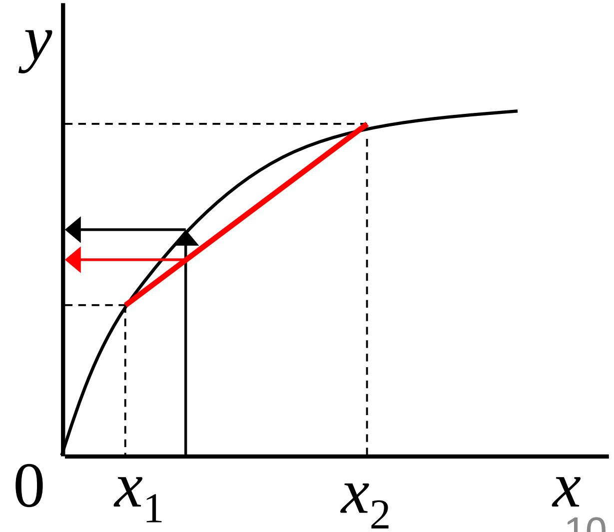
$\rho = 1$ $$f(x) = a_0 + a_1 x_1 + \cdots a_n x_n$$

Feature of Production Function (Assumption)

1. $f(0) = 0$
2. $f(x)$ is not monotonically decreasing with regard to x
3. $f(x)$ is a quasi-concave function (準凹関数)

   $\Leftrightarrow$ $V(y)$ is a convex set where

$$V(y) = \{x \in R_{+}^{n} | y \leq f(x)\}$$



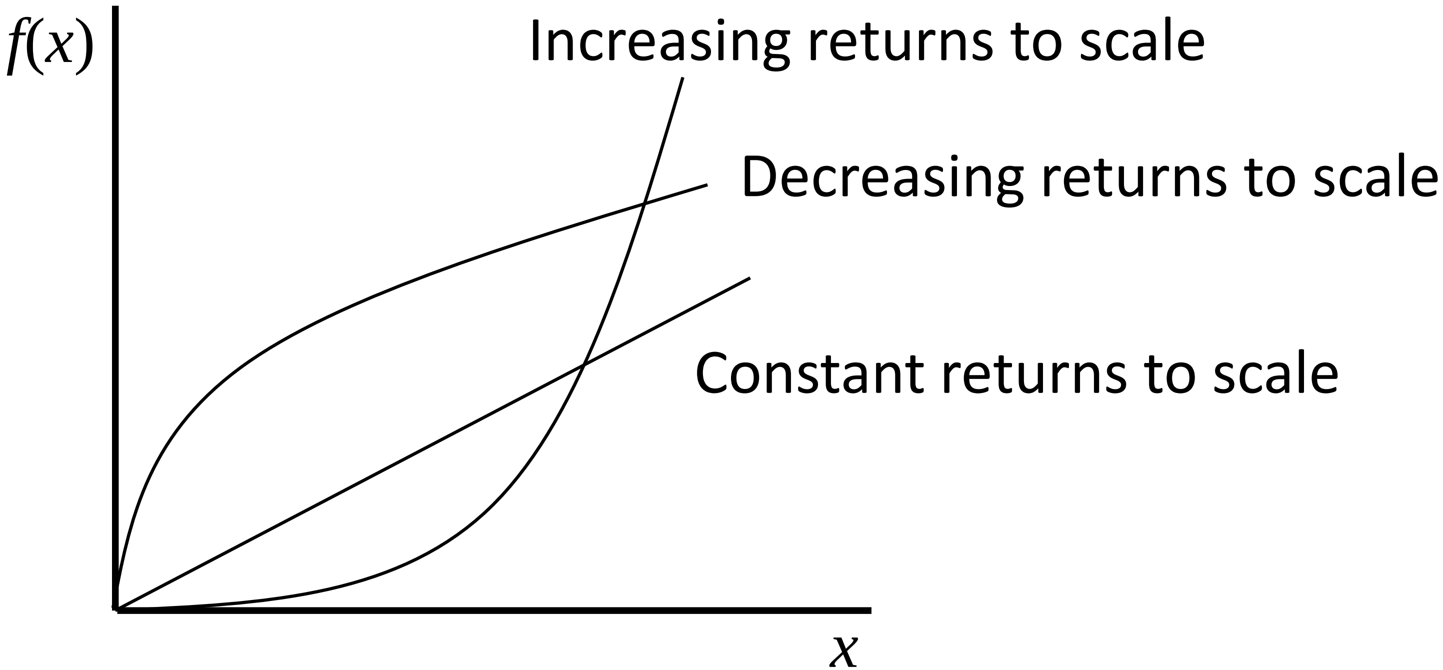
Returns to Scale

$$\forall x, x' \in R_+^n \ , 0 \le t \le 1$$

$$f\left(tx + (1-t)x'\right) \begin{cases} \le \\ = tf(x) + (1-t)f(x') \\ \ge \end{cases}$$

- Increasing returns to scale (規模に関して収穫逓増)
- Constant returns to scale (規模に関して収穫不変)
- Decreasing returns to scale (規模に関して収穫逓減)

Example of Returns to Scale



Firm’s behaviour

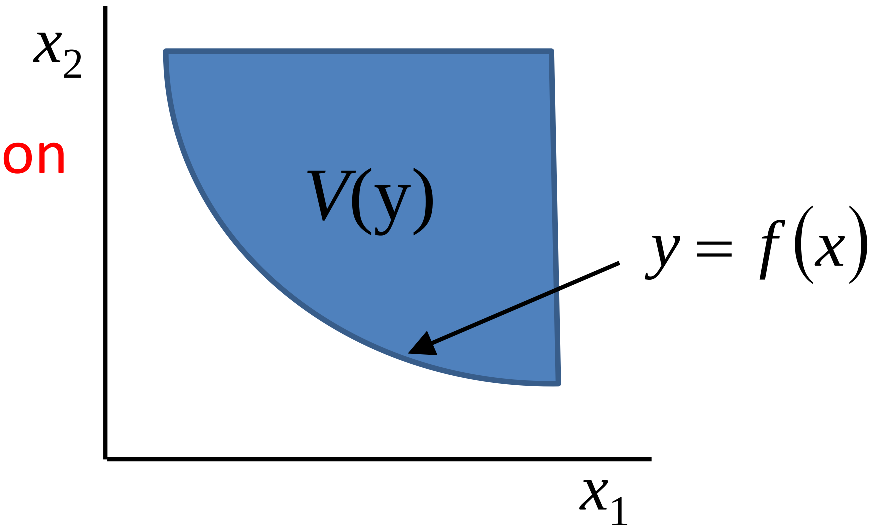
- Considering competitive firm(競争的企業)
- Profit maximisation

$$\max_{\mathbf{y},\mathbf{x}} \; py - \sum_{i=1}^{n} w_i x_i$$

Revenue   Cost

Such that

$y = f(x)$: Production function (生産関数)



Firm's behaviour (Cont.)

$$\pi(p,w) = \max_x \left[pf(x) - \sum_{i=1}^{n} w_i x_i \right]$$

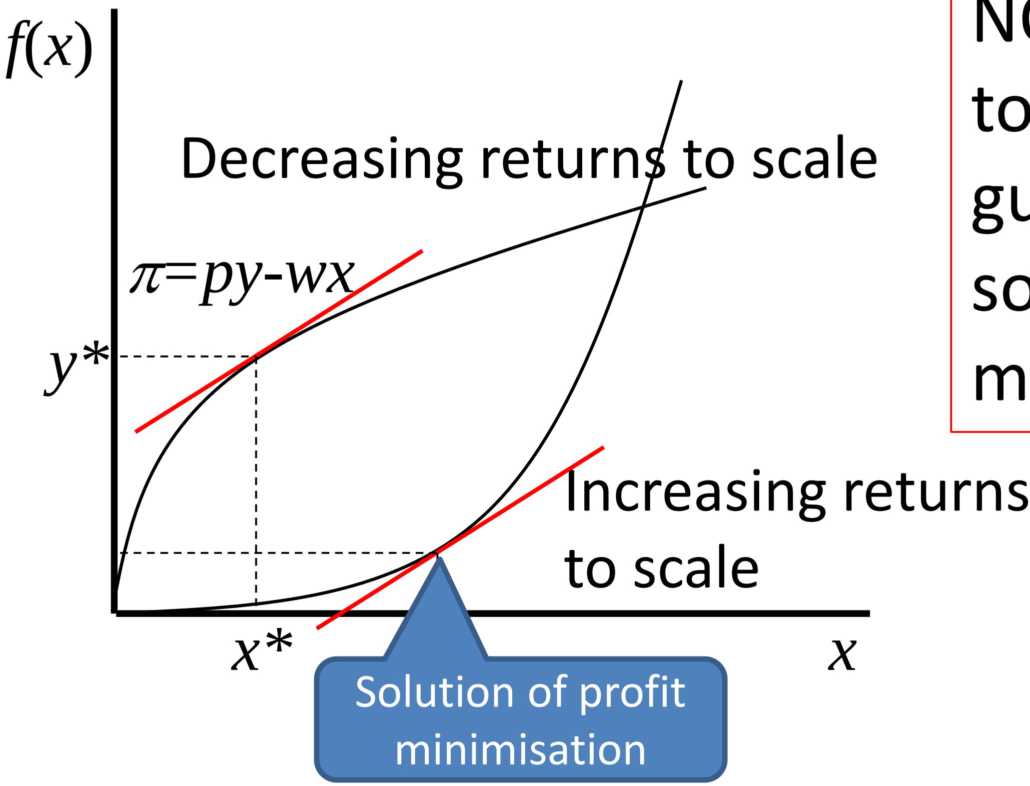
First order condition

$$p \frac{\partial f(x)}{\partial x_i} = w_i$$

Value of Marginal production = Factor price

限界生成物の価値　　要素価格

Returns to Scale and Profit Maximisation



Production function should NOT has increasing returns to scale in order to guarantee the existence of a solution of the profit maximisation problem.

* If production function is increasing returns to scale, the solution is $x = \infty$

Cost Minimisation

$$c(w,y) = \min \sum_{i=1}^{n} w_i x_i$$

Subject to $y = f(x)$

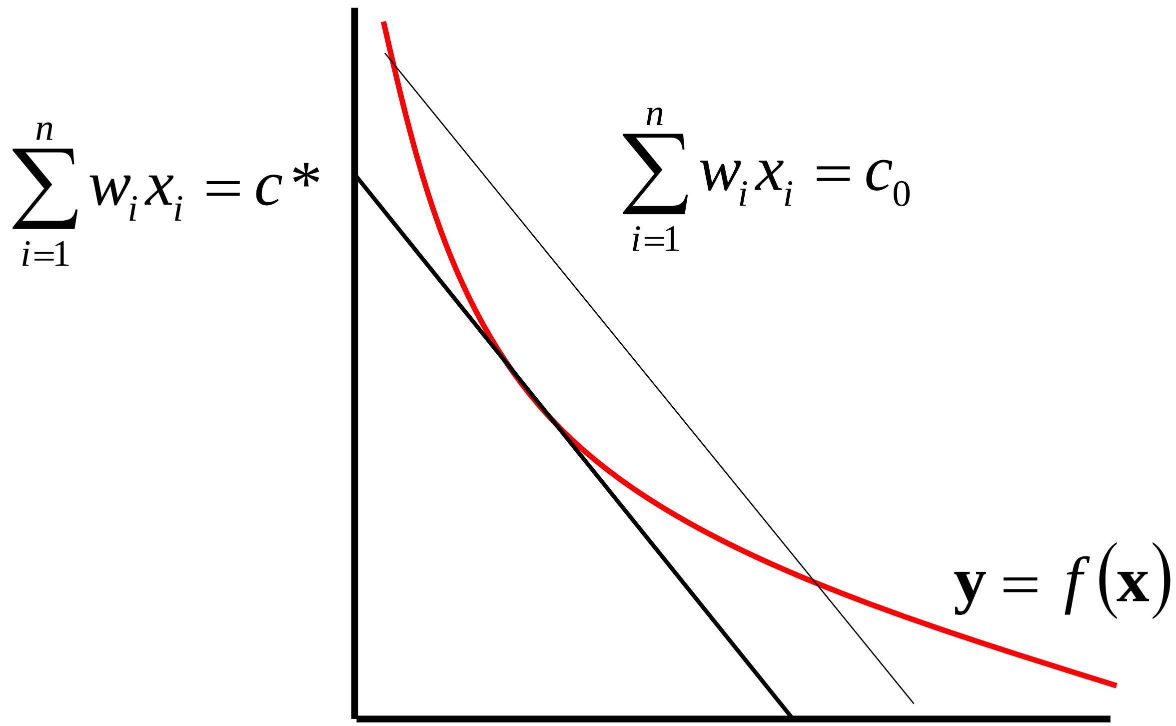
First order condition

$$\begin{cases} w_i = \lambda \dfrac{\partial f(x)}{\partial x_i} \\ y = f(x) \end{cases} \Rightarrow \; w_i / w_j = \frac{\partial f(x)}{\partial x_i} \Big/ \frac{\partial f(x)}{\partial x_j}$$

Factor price ratio = Technical marginal rate of substitution

要素価格比　　技術的限界代替率

Illustration



Conditional Factor Demand Functions
(条件付要素需要関数)

Solution of Cost Minimisation Problem;

$$x_i = x_i(w,y)$$

Shephard's Lemma (シェパードのレンマ)

$$x_i(w, y) = \frac{\partial c(w,y)}{\partial w_i}$$

Proof

By differentiating $c(w,y) = \sum_{i=1}^{n} w_i x_i(w,y)$ with respect to w_i,

$$\frac{\partial c(w,y)}{\partial w_i} = x_i(w,y) + \sum_{j=1}^{n} w_j \frac{\partial x_j(w,y)}{\partial w_i}.$$

From the First order condition, we know $w_i = \lambda \frac{\partial f(x)}{\partial x_i}$ holds.

Therefore, $$\frac{\partial c(w,y)}{\partial w_i} = x_i(w,y) + \sum_{j=1}^{n} \lambda \frac{\partial f(x)}{\partial x_i} \frac{\partial x_j(w,y)}{\partial w_i}$$

Furthermore, by differentiating $y = f(x(w,y))$ with respect to w_i,

$$0 = \sum_{j=1}^{n} \frac{\partial f(x)}{\partial x_j} \frac{\partial x_j(w,y)}{\partial w_i}$$

Therefore, $x_i(w,y) = \frac{\partial c(w,y)}{\partial w_i}$ [QED]

Profit Maximisation

$$\pi(p,w) = \max[py - c(w,y)]$$

First Order Condition

$$p = \frac{\partial c(w,y)}{\partial y}$$

Price

Marginal Cost

Factor Demand Function (要素需要関数)
Supply Function (供給関数)

Factor Demand Function $x_i = x_i(p, w)$

Supply Function $y = y(p, w)$

↓

Hotelling's Lemma (ホテリングのレンマ)

$$y(p, w) = \frac{\partial \pi(p, w)}{\partial p}$$

$$x_i(p, w) = -\frac{\partial \pi(p, w)}{\partial w_i}$$

Proof

$$\pi(p,w)=\max_{\mathbf{x}}\left[pf(x)-\sum_{i=1}^{n}w_i x_i\right] \Rightarrow p\frac{\partial f(x)}{\partial x_i}=w_i \quad \text{(First Order Condition)}$$

$$\pi(p,w)=pf(\mathbf{x})-\sum_i w_i x_i$$

$$\therefore \frac{\partial \pi}{\partial w_i} = \sum_j p\frac{\partial f}{\partial x_j}\frac{\partial x_j(p,w)}{\partial w_i}-\sum_j w_j\frac{\partial x_j(p,w)}{\partial w_i}-x_i$$

$$=\sum_j \underline{\left(p\frac{\partial f}{\partial x_j}-w_j\right)}\frac{\partial x_j(p,w)}{\partial w_i}-x_i$$

=0 ($\because\ p\frac{\partial f}{\partial x_i}=w_i$; First Order Condition)

$$\therefore x_i(p,w)=-\frac{\partial \pi(p,w)}{\partial w_i}$$

Proof (Cont.)

$$\pi(p,w) = \max[py - c(w,y)]$$

$$\begin{aligned}\frac{\partial \pi}{\partial p} &= y(p,w) - p\frac{\partial y}{\partial p} - \frac{\partial c}{\partial y}\frac{\partial y}{\partial p} \\ &= y(p,w) + \frac{\partial y}{\partial p}\underbrace{\left(p - \frac{\partial c}{\partial y}\right)}\end{aligned}$$

=0 ($\because$ $p = \frac{\partial c(w,y)}{\partial y}$; First Order Condition)

$$\therefore y(p,w) = \frac{\partial \pi(p,w)}{\partial p}$$

Short/Long-run Cost Function
(長期・短期の費用関数)

$$c(y) = c_v(y) + F$$

Cost (費用) = Variable Cost (可変費用) + Fixed Cost (固定費用)

- In the Short-run, some of the factors are fixed in production. → Short-run cost function has positive *F*.
- In the Long-run, no factors of production are fixed. → Long-run cost function has *F*=0.

Average Cost, Marginal Cost
(平均費用, 限界費用)

* We only consider short run cost

- Short-run Average Cost (AC) 短期平均費用

$$AC(y) = c(y)/y = c_v(y)/y + F/y$$

Short-run Average Variable Cost; AVC (短期平均可変費用)

Short-run Average Fixed Cost; AFC (短期平均固定費用)

- Short-run Marginal Cost (MC) 短期限界費用

$$MC(y) = \partial c(y)/\partial y = \partial c_v(y)/\partial y \quad (\because \partial F/\partial y = 0)$$

Average Cost, Marginal Cost (Cont)
(平均費用, 限界費用)

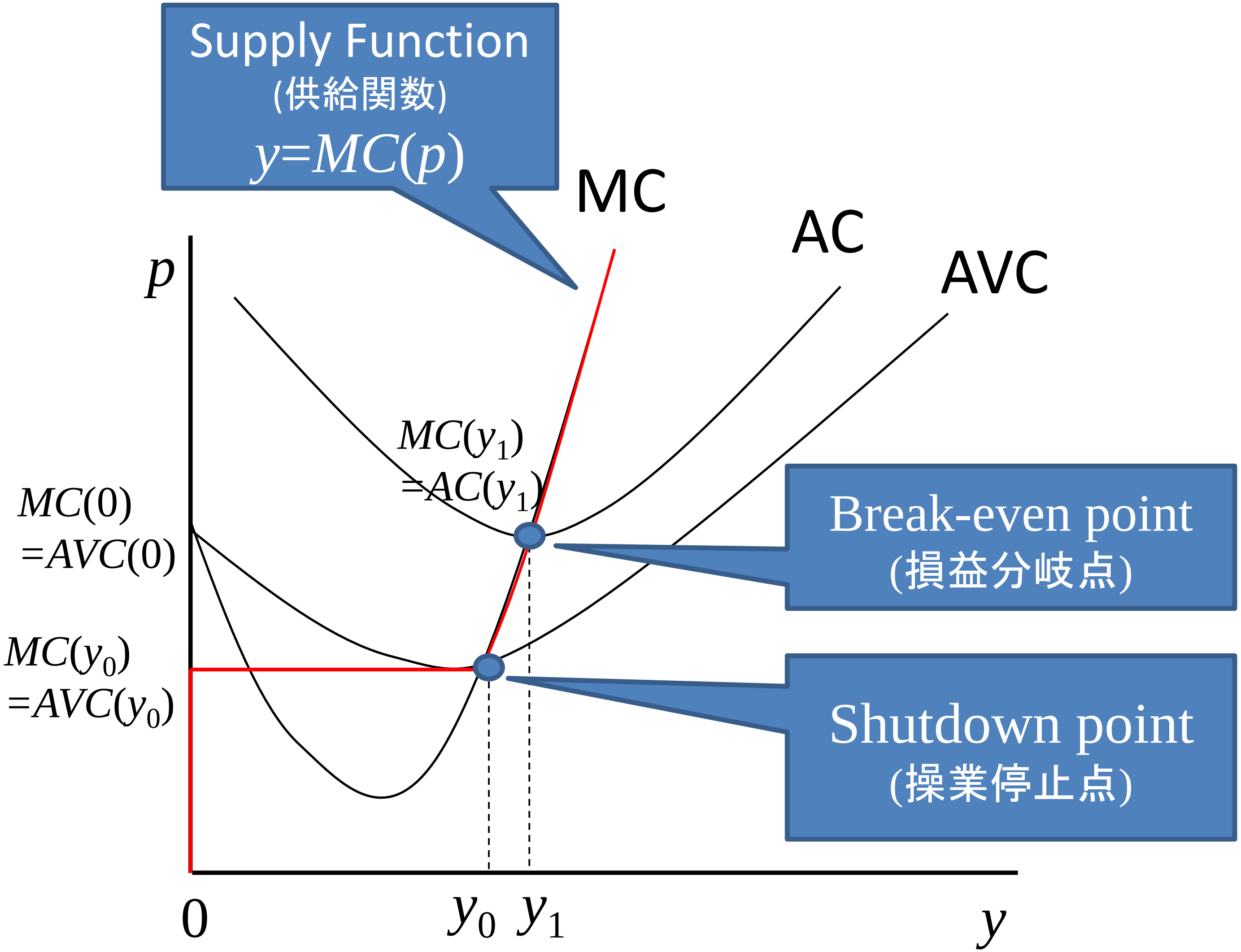
Marginal Cost curve satisfies following properties;

1. $MC(0) = AVC(0)$
2. MC curve must intersect with the AVC curve at its minimum point
3. MC curve must intersect with the AC curve at its minimum point

* Proof of property 3;

$$\min_y AC(y) = \min_y \frac{c(y)}{y} \Rightarrow \frac{\partial c(y)/y - c(y)}{y^2} = 0 \Leftrightarrow \frac{c(y)}{y} = \frac{\partial c(y)}{\partial y}$$

Illustration



Break-even point, Shutdown point and Supply Function

- Break-even point (損益分岐点)
  - Combination of price (p) and the amount of production(y) at which the firm's profit is zero.

$$py - c(y) = 0 \Rightarrow p = c(y)/y = AC(y)$$

- Shutdown point (操業停止点)
  - Combination of price (p) and the amount of production(y) where the a firm is indifferent between continuing operations and shutting down temporarily.

$$py - (c_v(y) + F) \geq 0 - (c_v(0) - F) \Rightarrow p \geq c_v(y)/y = AVC(y)$$

- Supply Function (供給曲線)
  - Solution of Max py-$c(y)$ ⇒

$$\begin{cases} y = MC^{-1}(p) & (p \geq \min AVC(y)) \\ y = 0 & (p < \min AVC(y)) \end{cases}$$