Theory of Consumer Behaviour

What is Consumer Behaviour?

- Suppose you earn 12,000 yen additionally
  - How many times do you enjoy lunch with 1,000 yen (x_1) and how many times do you watch movie with 2,000 yen (x_2)?

(x_1, x_2) = (10,1), (6,3), (3,4), (2,5), …

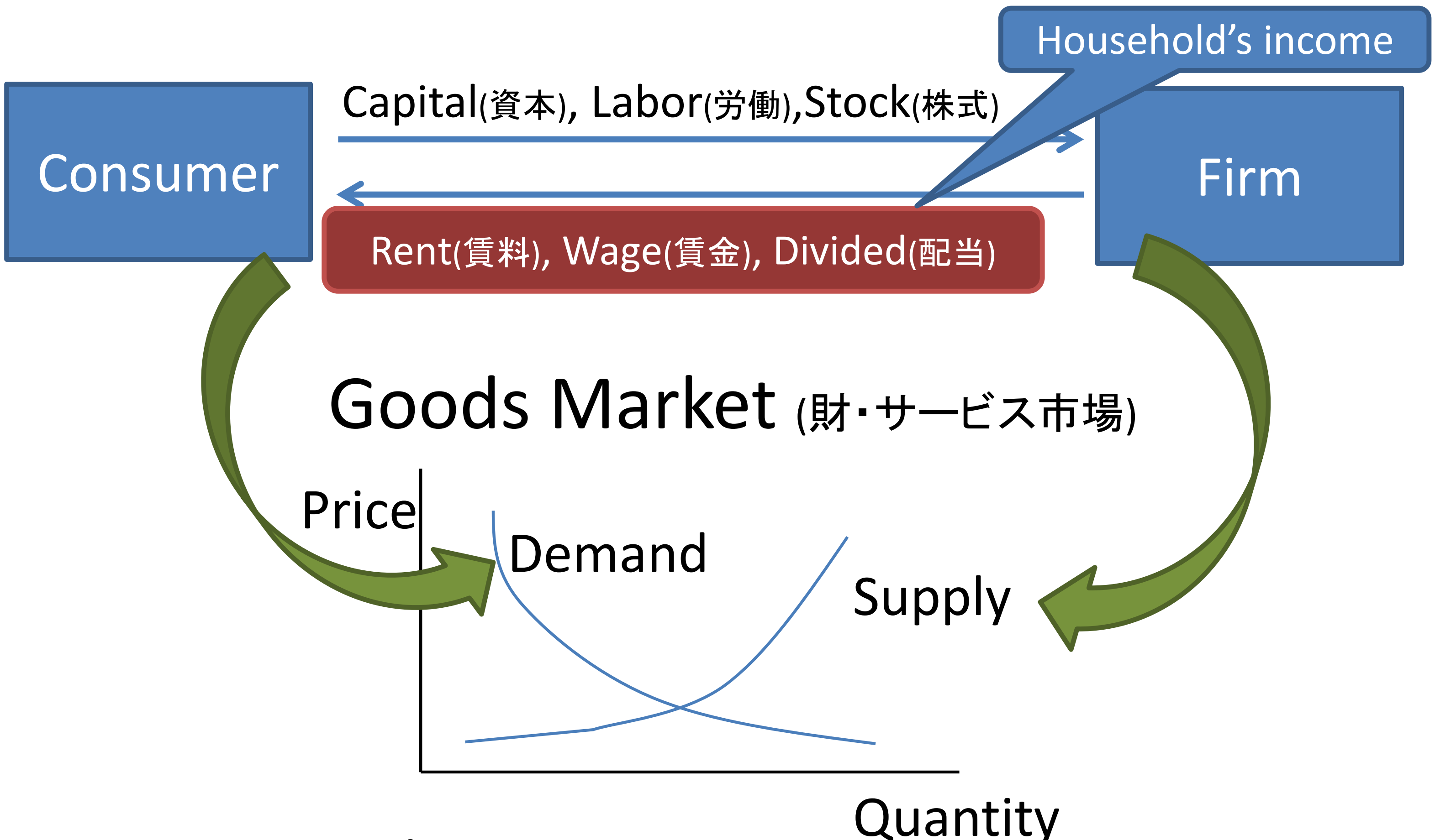
  - Suppose the price of movie is 1,500 yen?
  - Suppose the additional bonus is 10,000 yen?

Consumer Behaviour

- Feature of Consumer Behaviour
- Consumption set (Budget constraint)
- Preference
- Utility
- Choice
- Demand
- Revealed preference

Feature of Consumer Behaviour

Economic Entity (経済主体) → Firm(企業), Consumer (家計), Government



Consumer = price taker (価格受容者)

Budget Set (1)

- Constraint faced by consumer
  - Budget Constraint (income is limited)
  - Time Constraint (time is limited)
  - Allocation Constraint

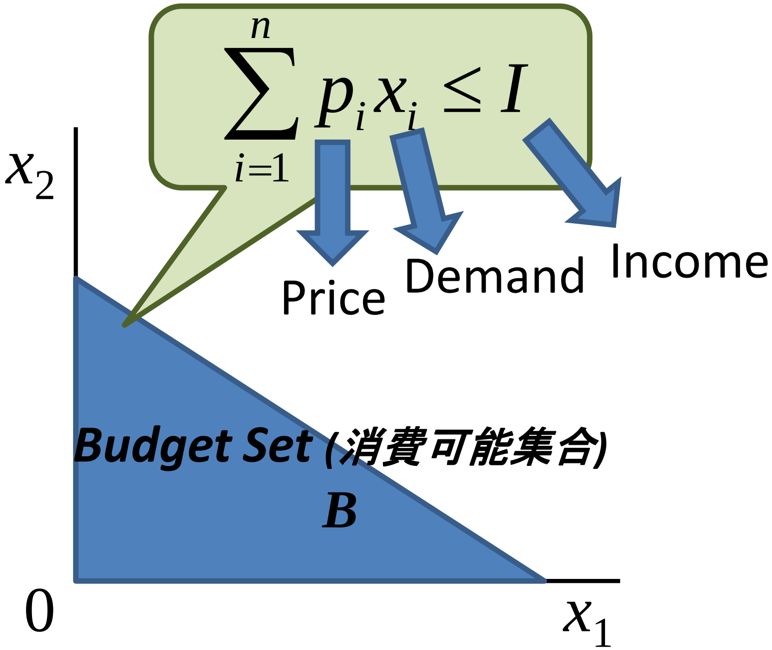
Possible to convert into monetary unit under the given wage rate

Combine to Budget Constraint

Generally, only the budget constraint is considered

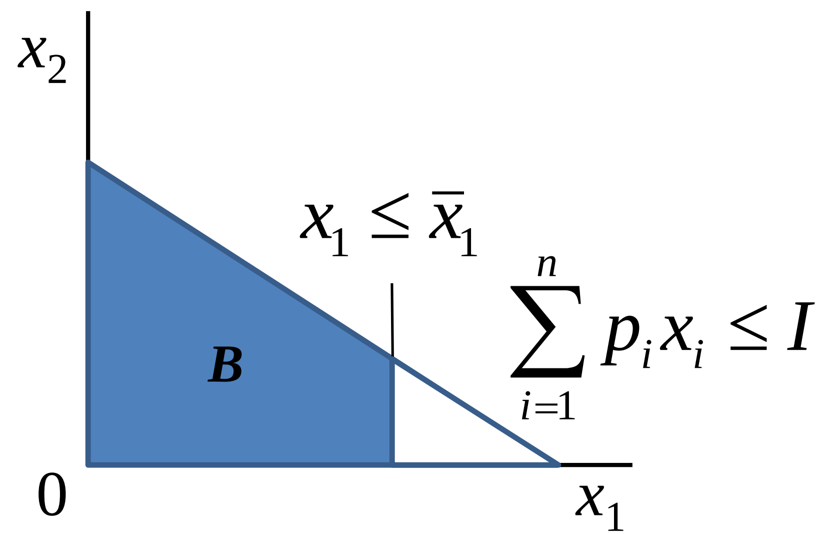
Budget Set (2)

Budget Constraint
(without allocation constraint)



$$B = \left\{ x \in R^n \middle| x \geq 0, \sum_{i=1}^{n} p_i x_i \leq I \right\}$$

Budget Constraint
(with allocation constraint)



$$B = \left\{ x \in R^n \middle| x \geq 0, \sum_{i=1}^{n} p_i x_i \leq I, x_1 \leq \bar{x}_1 \right\}$$

Preference (1)

- What is preference?

$A \succ B$ ⟺ A is (strictly) preferred to B
(A is always chosen between A and B)

$A \succeq B$ ⟺ A is preferred to B, or indifferent between two
(B is never chosen between A and B)

$A \sim B$ ⟺ A and B is indifferent
(No difference between A and B)

Preference (2)

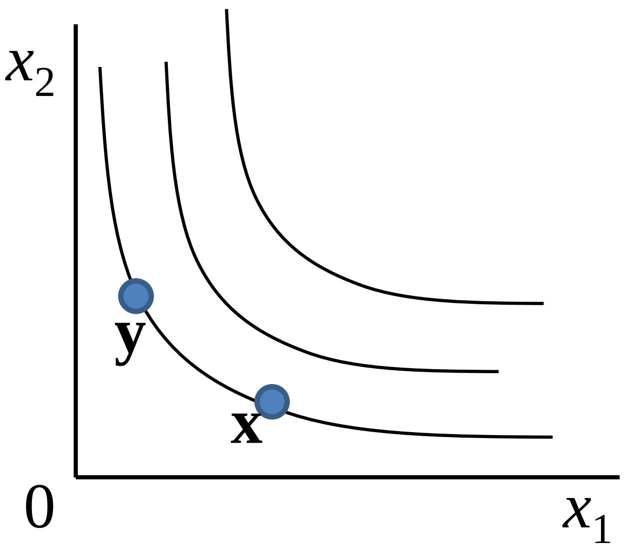
- Assumption regarding to preference
  1. Complete: Either $A \succ B$, $A \succeq B$ or $A \sim B$ is satisfied

     (完備性or完全性)
  2. Transitive : $A \succ B$ and $B \succ C$ then $A \succ C$

     (推移性)
  3. Reflexive : $A \succeq A$

     (連続性or反射性)

Preference (3) – indifference curve

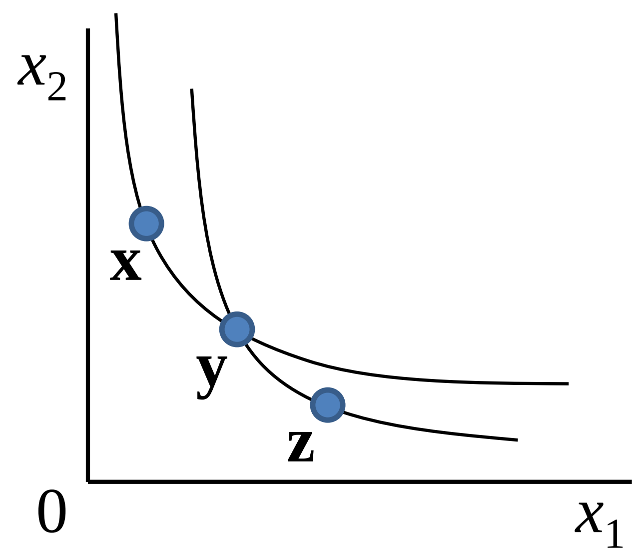


Indifference curve (無差別曲線)

$$C(\mathbf{x}) = \left\{ \mathbf{y} \in R^n \middle| \mathbf{y} \sim \mathbf{x} \right\}$$

Question

Are these two lines satisfy the three assumptions?

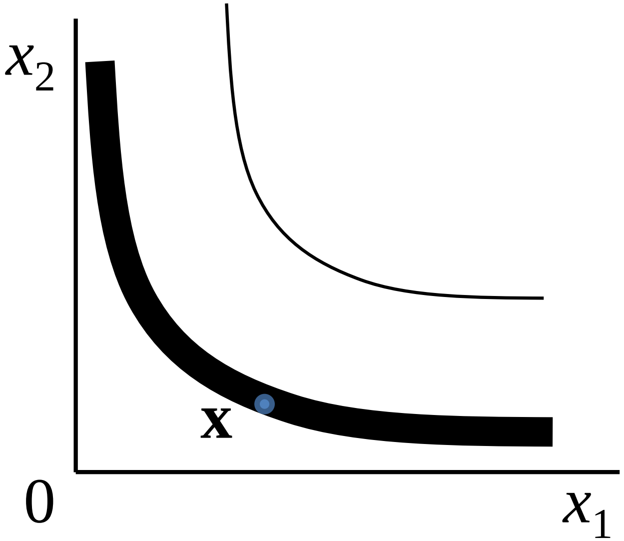


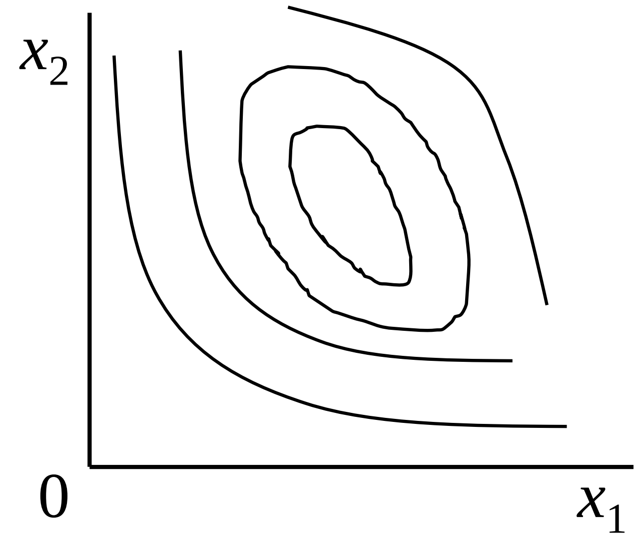
x, **y** and **z** would be indifferent, which is obviously inconsistent

Preference (4) – indifference curve

Question

Are these two lines satisfy the three assumptions?

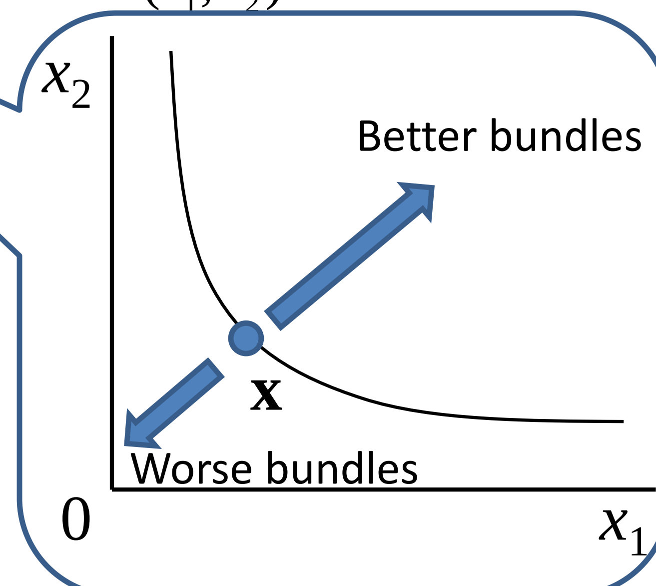




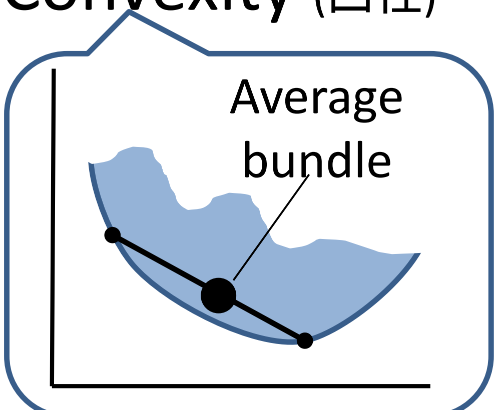
Preference (5)

- Additional assumptions regarding to preference

1. Monotonicity (単調性): $x_1 > x_1' \Rightarrow A \succ B$ $\quad A(x_1, x_2)$, $B(x_1', x_2)$



2. Convexity (凸性)



\+ Complete, Transitive and Reflexive

→ There exist Utility Function

Utility Function

- What is utility function?

Definition

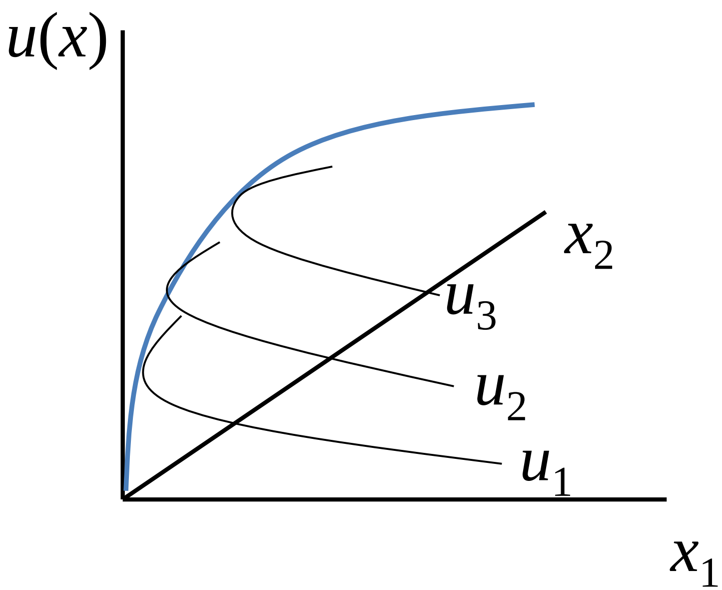
$\forall \mathbf{x}, \mathbf{y} \subseteq R^n, \mathbf{x} \succ \mathbf{y} \Leftrightarrow$ There exist $u: R^n \rightarrow R$ that satisfies $\boldsymbol{u(\mathbf{x}) \geq u(\mathbf{y})}$

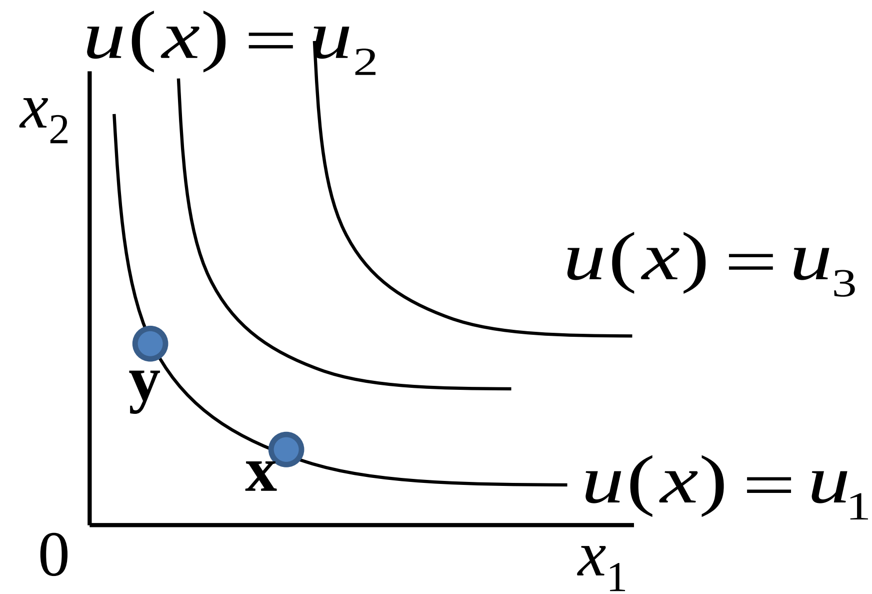
Theorem

If the preference satisfies **complete**, **transitive**, **reflexive** and **monotonicity**, then there exist utility function that satisfies

$\forall \mathbf{x}, \mathbf{y} \subseteq R^n, \mathbf{x} \succ \mathbf{y} \Leftrightarrow$ There exist $u: R^n \rightarrow R$ that satisfies $\boldsymbol{u(\mathbf{x}) > u(\mathbf{y})}$

Utility function and Indifference curve

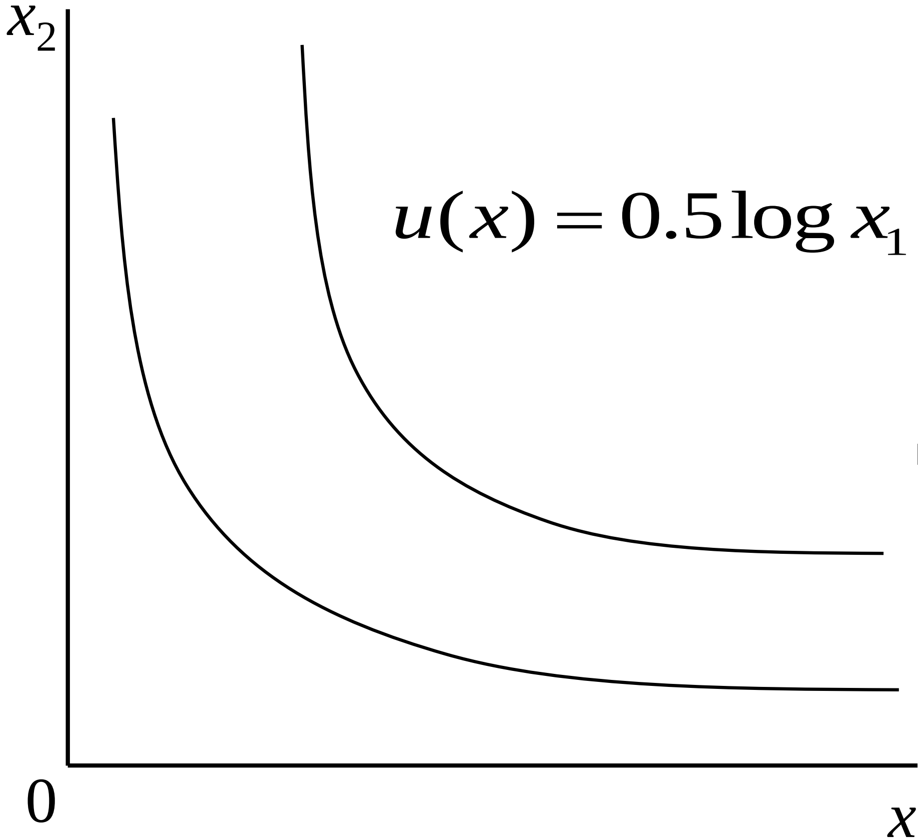


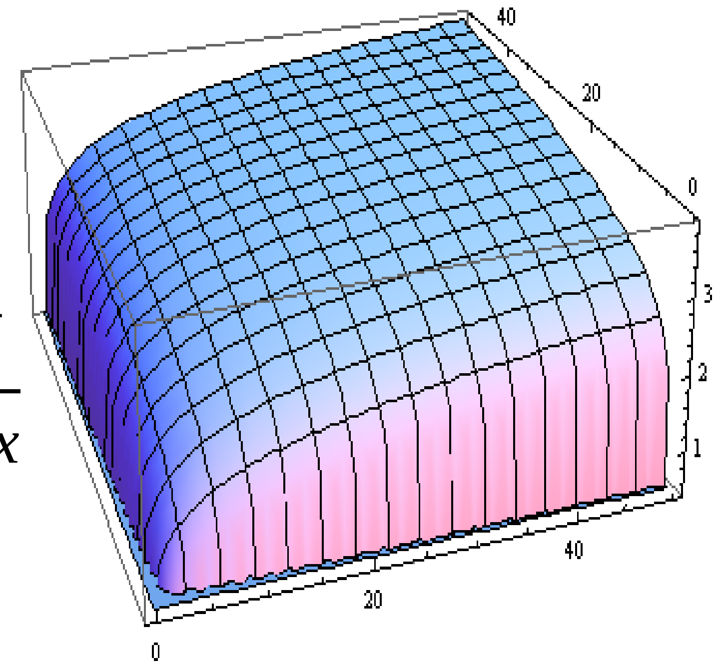


Indifference curve is expressed as a **contour line** (等高線) of an **utility function**

Example of utility function

$$u(x) = 0.5\log x_1 + 0.5\log x_2$$





Various Utility Function

- Ordinal utility (序数的効用)

  Only the order of the utilities is meaningful

  - Cobb-Douglas type $\begin{cases} u(x_1, x_2) = x_1^{\alpha} x_2^{\beta} \text{ or} \\ u(x_1, x_2) = \alpha \ln x_1 + \beta \ln x_2 \end{cases}$
  - Linear $u(x_1, x_2) = \alpha x_1 + \beta x_2$
  - Leontief type $u(x_1, x_2) = \min[\alpha x_1, \beta x_2]$
  - CES type $u(x_1, x_2) = \left(a x_1^{\rho} + b x_2^{\rho}\right)^{-\rho}$

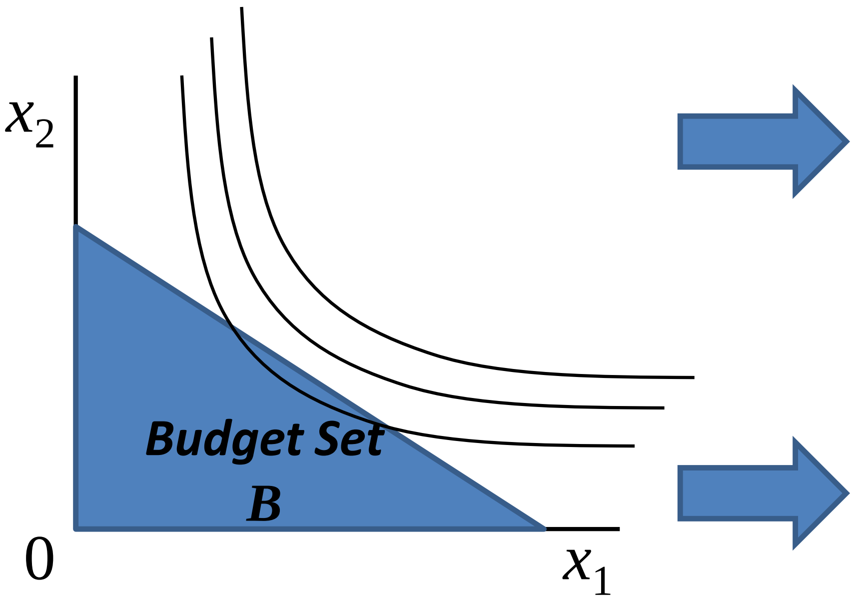
- Cardinal Utility (基数的効用)

  The value of the utilities is also meaningful

Can we express the value of utility correctly?

Choice (選択)

- Consumers are assumed to choose most preferable bundle from their budget set



The utility of the right upper indifference curve is higher

We should find a bundle whose utility is maximum among a given budget set

Consumer Behaviour Model

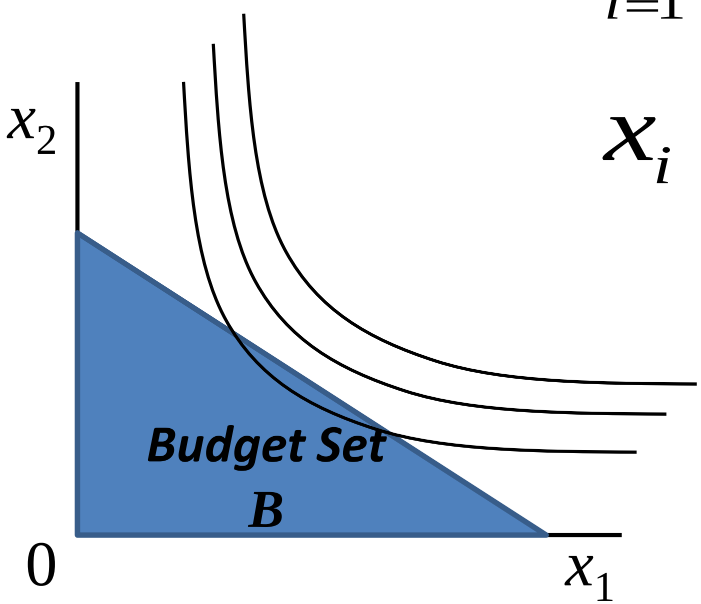
(消費者行動モデル)

$$\max_{x} u(x_1, x_2, \ldots, x_n)$$

subject to

$$\sum_{i=1}^{n} p_i x_i \leq I$$

$$x_i \geq 0 \; (i = 1, \ldots, n)$$



Monotonical utility function

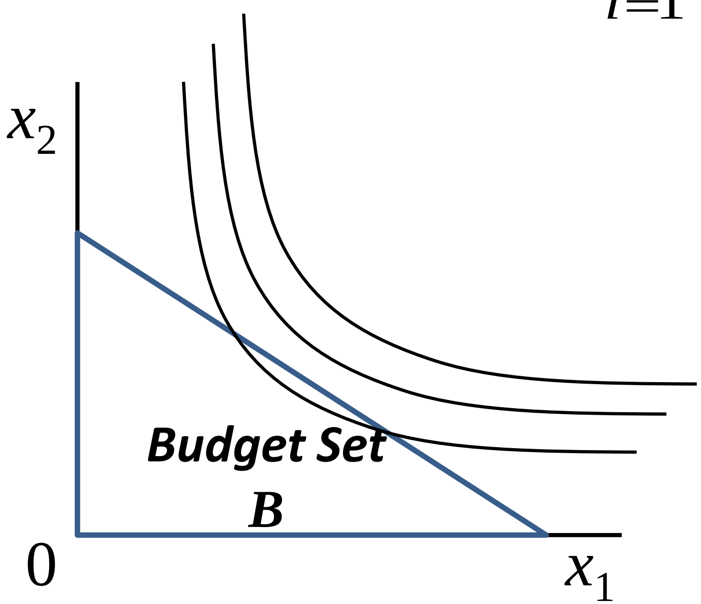
Consumer Behaviour Model

(消費者行動モデル)

$$\max_{x} u(x_1, x_2, \ldots, x_n)$$

subject to

$$\sum_{i=1}^{n} p_i x_i = I$$



First order condition
(一階条件)

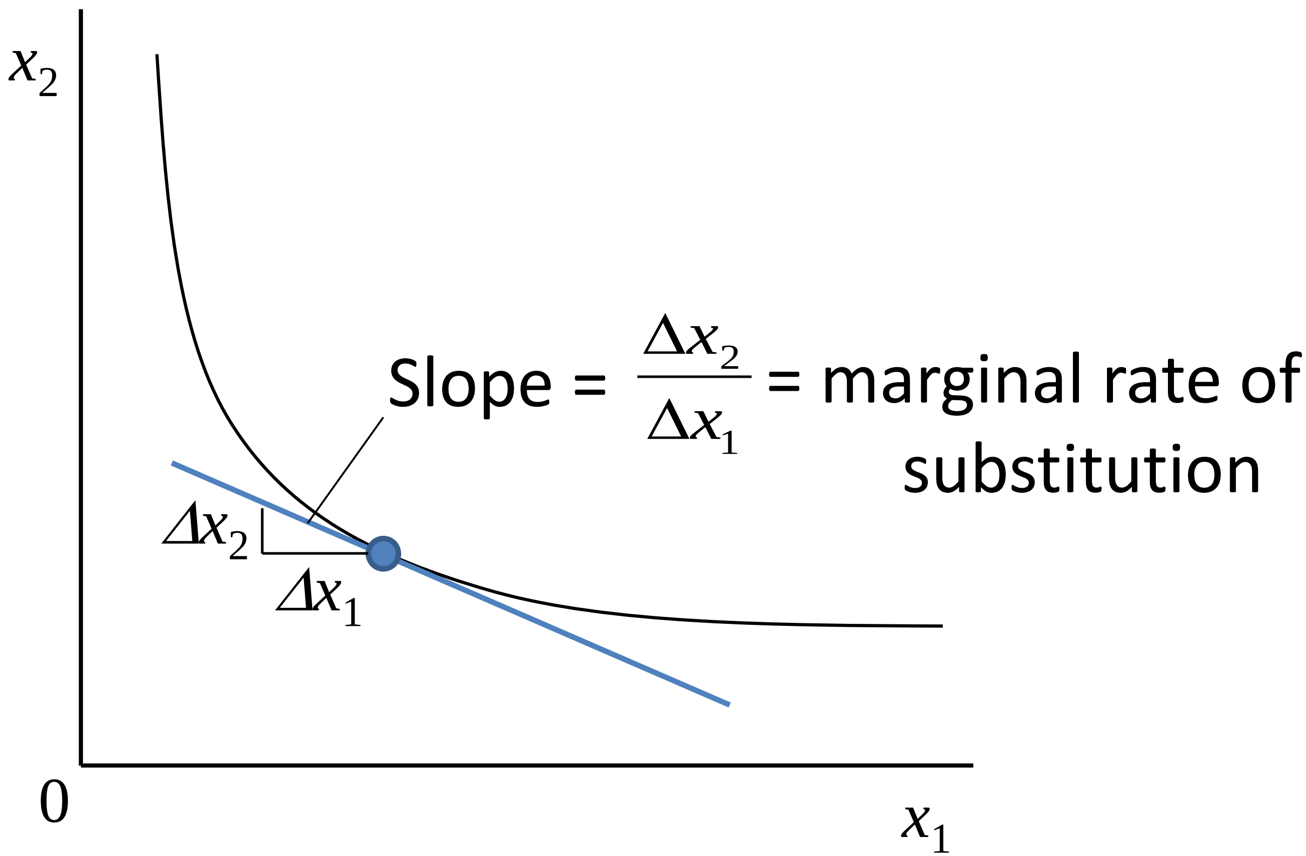
$$L(x,\lambda) = u(x_1, x_2, \ldots, x_n) - \lambda\left(\sum_{i=1}^{n} p_i x_i - I\right)$$

⇓

$$\partial L/\partial x_i = 0: \quad \partial u/\partial x_i = \lambda p_i$$

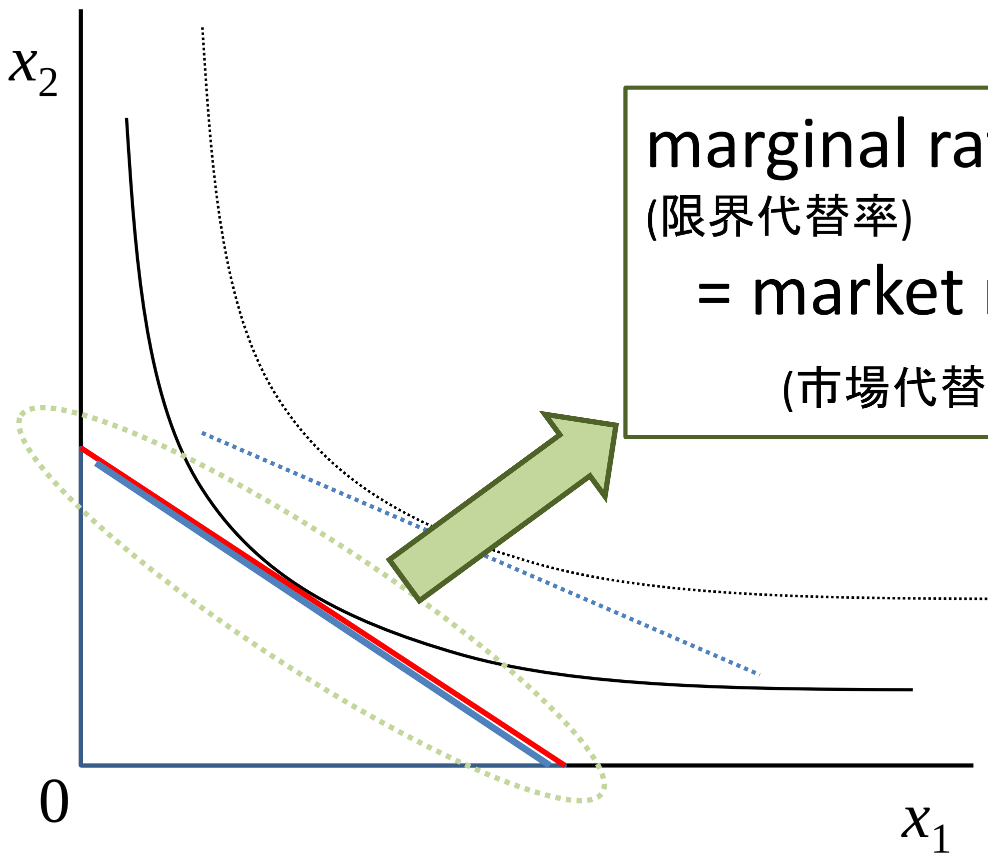
$$\partial L/\partial \lambda = 0: \quad \sum_{i=1}^{n} p_i x_i = I$$

Marginal Rate of Substitution
(MRS;限界代替率)



MRS measures the rate at which the consumer is just willing to substitute one good for the other

Graphic illustration of first order condition



marginal rate of substitution
(限界代替率)
= market rate of substitution
(市場代替率)

$$\frac{\partial u / \partial x_i}{\partial u / \partial x_j} = \frac{p_i}{p_j}$$

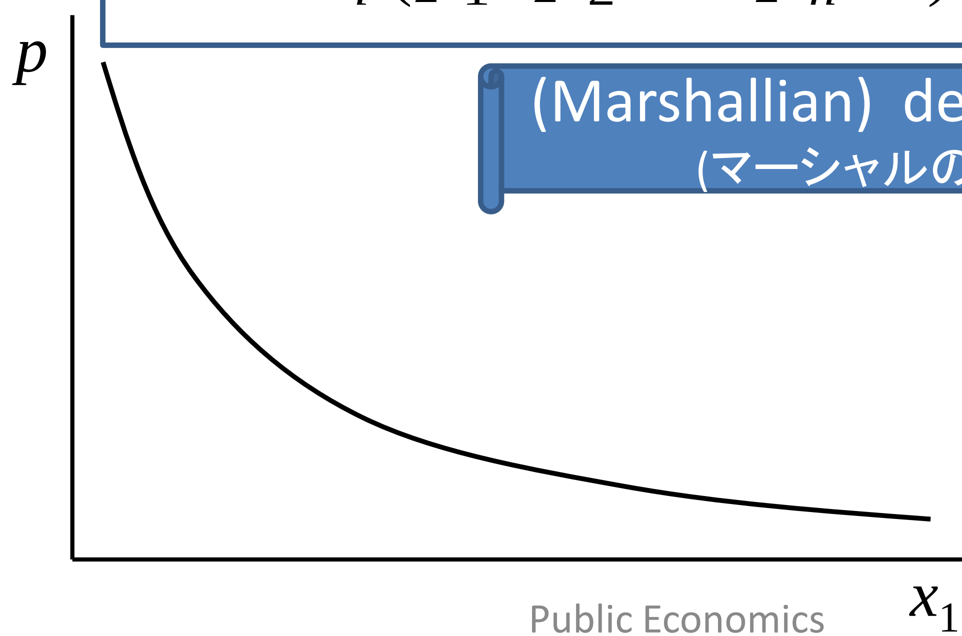
Demand (需要)

- The **consumer's demand function** give the optimal amount of each of the goods as a **function of** the **prices** and **income** faced by the consumer

Solution of the Consumer behaviour model

$$x_i(p_1, p_2, \ldots, p_n, I)$$

(Marshallian) demand function
(マーシャルの)需要関数



Example

- Find demand function with Cobb-Douglas type utility function

$$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta} \to \max$$

subject to

$$p_1 x_1 + p_2 x_2 = I$$

Hint:
It is easier to solve if we assume Cobb-Douglas type utility function as $u(x_1, x_2) = \alpha \ln x_1 + \beta \ln x_2$

Answer

- First order condition

$$L = a\ln x_1 + b\ln x_2 - \lambda\left(p_1x_1 + p_2x_2 - I\right)$$

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = \dfrac{a}{x_1} - \lambda p_1 = 0 \\ \dfrac{\partial L}{\partial x_2} = \dfrac{b}{x_2} - \lambda p_2 = 0 \end{cases}$$

$$\frac{\partial L}{\partial \lambda} = p_1x_1 + p_2x_2 - I = 0$$

- Rewrite above equations using MRS

$$\begin{cases} \dfrac{a/x_1}{b/x_2} = \dfrac{p_1}{p_2} \\ p_1x_1 + p_2x_2 - I = 0 \end{cases}$$

Answer (Cont)

- By solving those equations

$$
\begin{cases}
x_1(p_1, p_2, I) = \dfrac{a}{a+b}\dfrac{I}{p_1} \\
x_2(p_1, p_2, I) = \dfrac{b}{a+b}\dfrac{I}{p_2}
\end{cases}
$$

Homogeneous function

Definition

If a function $f(x)$ satisfies following feature, then f is said to be **homogeneous** of degree k;

(k次同次関数)

$$\forall t > 0, \mathbf{x} \in R^n,$$

$$f(t\mathbf{x}) = t^k f(\mathbf{x})$$

Question

Assume Cobb-Douglas type utility function and proof following propositions

1. Demand function is homogeneous with degree 0
2. Demand function is monotonic decrease (単調減少) with regard to price and monotonic increase (単調増加) with regard to income

Answer

$$x_1(p_1, p_2, I) = \frac{a}{a+b}\frac{I}{p_1}$$

1. $$x_1(tp_1, tp_2, tI) = \frac{a}{a+b}\frac{tI}{tp_1} = x_1(p_1, p_2, I)$$

2. It can easily be shown from above demand function

Indirect Utility Function (間接効用関数)

- A consumer's **indirect utility function** $v(\mathbf{p},I)$ gives the consumer's maximal utility when faced with a price $\mathbf{p}$ and an amount income I. It represents the consumer's preference over market conditions.

$$v(p_1,\ldots,p_n,I)=\max_{\mathbf{x}} u(x_1,\ldots,x_n)$$

subject to $\sum_{i=1}^{n} p_i x_i = I$

Identity (恒等式)

$$v(p_1,\ldots,p_n,I)=u(x_1(p_1,\ldots,p_n,I),\ldots,x_n(p_1,\ldots,p_n,I))$$

Example

1. Find indirect utility function whose utility function is Cobb-Douglas type

2. Proof following identity between indirect utility function and demand function

$$x_1(p_1,\ldots,p_n,I)=\frac{-\partial v(p_1,\ldots,p_n,I)/\partial p_i}{\partial v(p_1,\ldots,p_n,I)/\partial I}$$

Roy's identity (ロイの恒等式)

Answer

1. From the identify,

$$v(p_1, p_2, I) = \left(\frac{a}{a+b}\frac{I}{p_1}\right)^a \left(\frac{b}{a+b}\frac{I}{p_2}\right)^b = \frac{a^a b^b}{(a+b)^{a+b}}\frac{I^{a+b}}{p_1^a p_2^b}$$

2. (In case n=2)

We can get derivative of indirect utility function as following; $\frac{\partial v}{\partial p_1} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial p_1} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial p_1}$ and $\frac{\partial v}{\partial I} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial I} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial I}$

Then, if we differentiate the budget constraint ($p_1 x_1 + p_2 x_2 = I$) with respect to p_1 (with fixed I), we can get $x_1 + p_1\frac{\partial x_1}{\partial p_1} + p_2\frac{\partial x_2}{\partial p_2} = 0$

Also, by differentiating by I, we can get $p_1\frac{\partial x_1}{\partial I} + p_2\frac{\partial x_2}{\partial I} = 1$

From those equations and the condition of utility maximisation ($(\partial u/\partial x_1)/(\partial u/\partial x_2) = p_1/p_2$), we can obtain

$$x_1(p_1, p_2, I) = -\frac{\partial v/\partial p_1}{\partial v/\partial I}$$

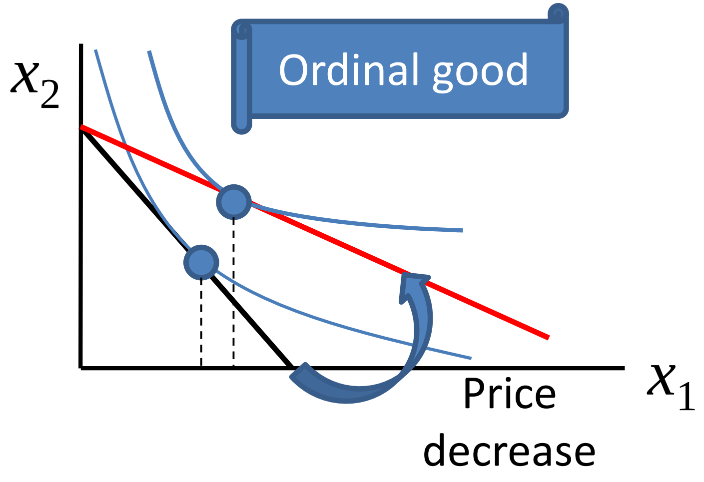
Q.E.D.

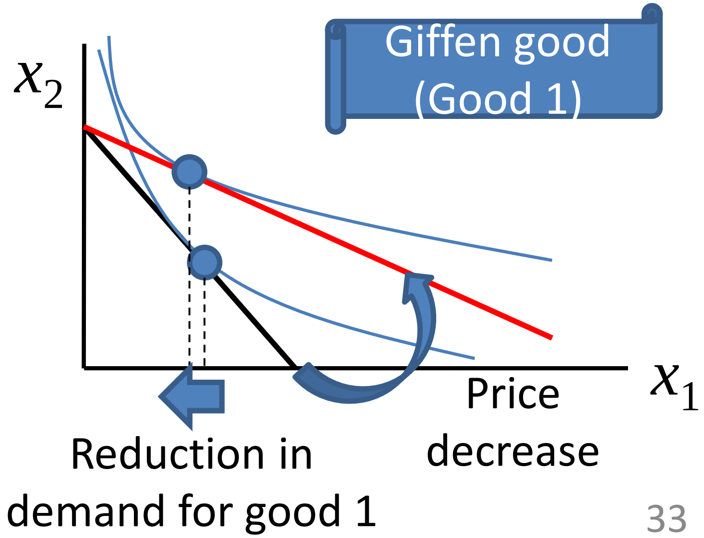
Income change and Demand

- Superior good (上級財)
  - Good whose demand increases as income increase
  - Example: Luxury goods
- Intermediate good (中間財)
  - Good whose demand is stable against income
  - Example: Tissue paper
- Inferior good (下級財)
  - Good whose demand decreases as income increase
  - Example: Substitution of rice (such as potato)

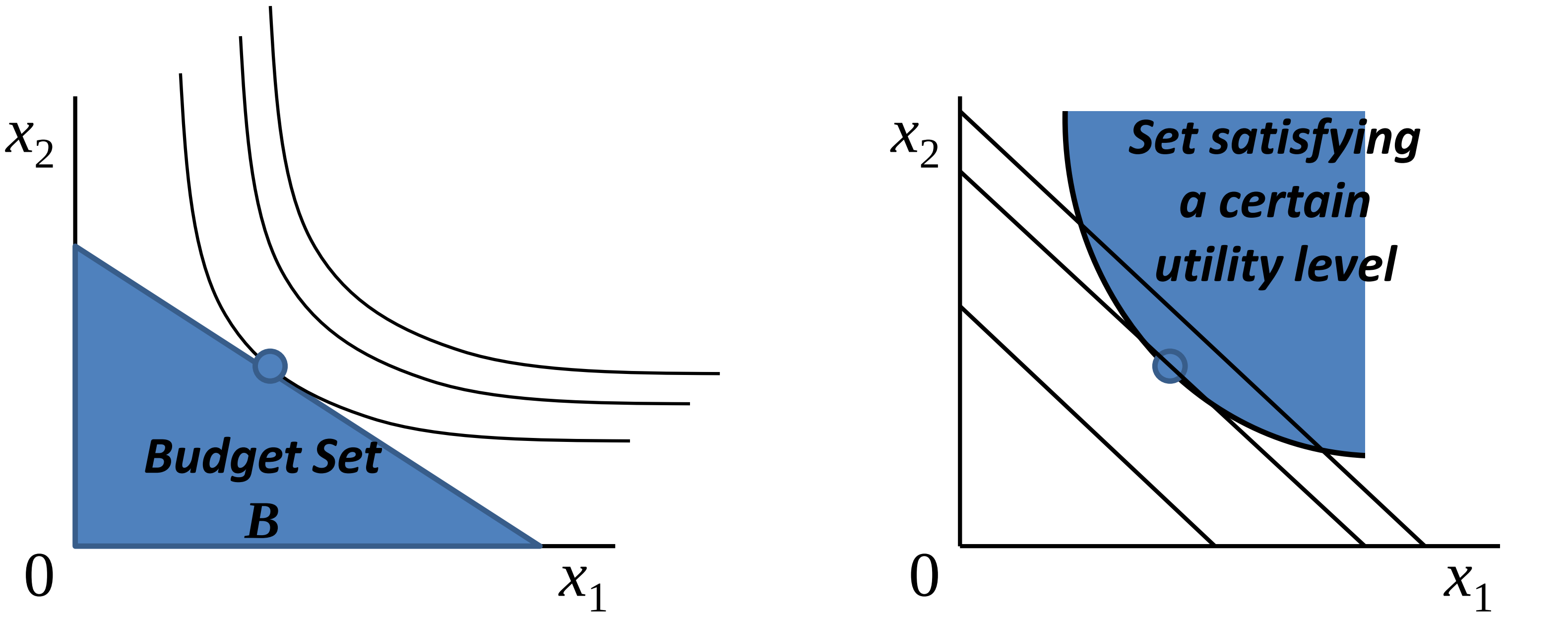
Price change and Demand

- Ordinal good (正常財)
  - Good whose demand decrease as it’s price increase
  - Example: Beer
- Giffen good (ギッフェン財)
  - Good whose demand increase as it’s price increase
  - Example: Substitution of rice (such as potato)





Another approach describing Consumer Behaviour



Expenditure Minimisation Problem
(支出最小化問題)

$$\min_{\mathbf{x}} \sum_{i=1}^{n} p_i x_i$$

subject to

$$u(x_1, \ldots, x_n) \geq \underline{u}$$

- Expenditure function (支出関数) $e(\mathbf{p}, \underline{u})$
- Hicksian demand (ヒックスの需要関数) $h(\mathbf{p}, \underline{u})$

Expenditure Function

Expenditure Minimisation Problem

$$e(p_1,\ldots,p_n,\underline{u}) = \min_{\mathbf{x}} \sum_{i=1}^{n} p_i x_i$$

Expenditure Function

subject to $u(x_1,\ldots,x_n) \geq \underline{u}$

First Order Condition

$$\frac{p_i}{p_j} = \frac{\partial u(x_1,\ldots,x_n)/\partial x_i}{\partial u(x_1,\ldots,x_n)/\partial x_j}$$

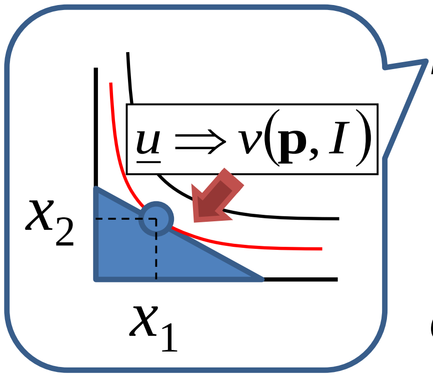
$$u(x_1,\ldots,x_n) = \underline{u}$$

Hicksian Demand Function

- Solution of Expenditure Minimisation Problem

$$h_i(\mathbf{p}, \underline{u})$$

- Identity (恒等式)

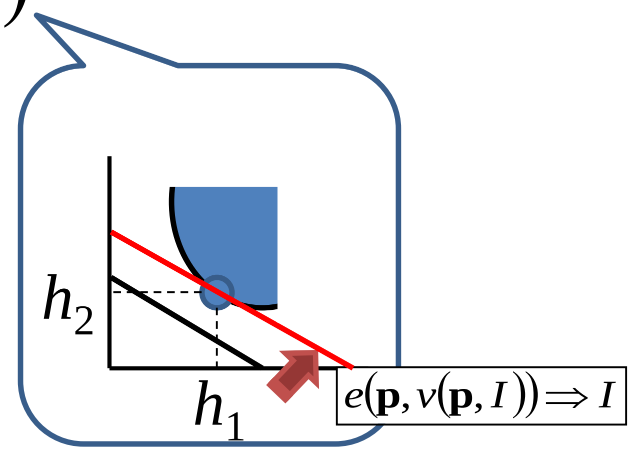


$$h_i(\mathbf{p}, v(\mathbf{p}, I)) = x_i(\mathbf{p}, I)$$

$$x_i(\mathbf{p}, e(\mathbf{p}, I)) = h_i(\mathbf{p}, \underline{u})$$

$$e(\mathbf{p}, v(\mathbf{p}, I)) = I$$

$$v(\mathbf{p}, e(\mathbf{p}, \underline{u})) = \underline{u}$$



Feature of Expenditure Function and Hicksian Demand Function

- Expenditure Function ($e(\mathbf{p}, u)$) is homogenous of degree 1 with regard to p. Expenditure Function is increasing function with regard to p and u.
- Identity (恒等式)

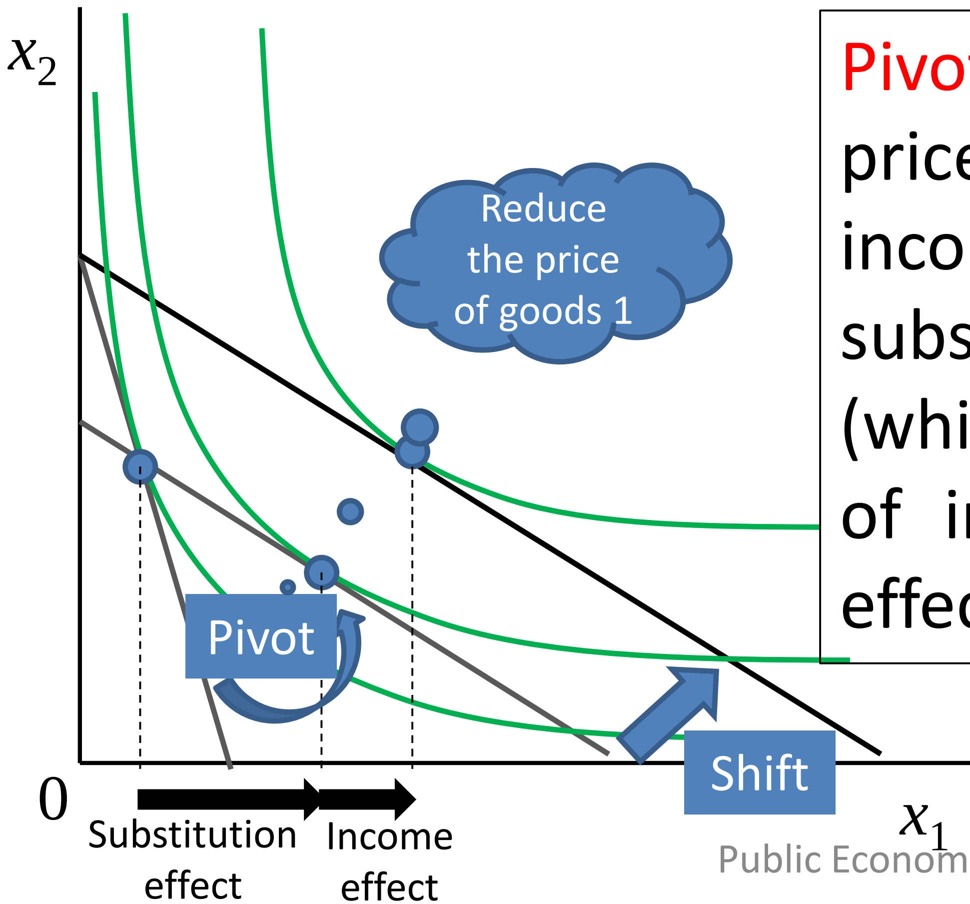
$$e(\mathbf{p}, \underline{u}) = \sum_{i=1}^{n} p_i h_i(\mathbf{p}, \underline{u})$$

$$h_i(\mathbf{p}, \underline{u}) = \frac{\partial e(\mathbf{p}, \underline{u})}{\partial p_i}$$

Income Effect and Substitution Effect

(所得効果と代替効果)

- **The effect of changing the price of goods**
  = Income Effect + Substitution Effect



Pivot (which represents the price of goods 1 change but income stays fix) gives the substitution effect, and Shift (which represents the change of income) gives the income effect

Income Effect and Substitution Effect
(所得効果と代替効果)

- **Substitution effect**… the change in demand due to the change in rate of exchange between two goods

  If the price of good 1 decreases,
  the price of good 2 increases relatively

- **Income effect** … the change in demand due to having more purchasing power

  If the price of good 1 decrease,
  the substantive income will increase

SLUTSKY Equation
(スルツキー方程式)

- Equation representing the relationship between **Income Effect** and **Substitution Effect**
(i.e. **The effect of changing the price of goods**
= **Income Effect** + **Substitution Effect**)

$$\frac{\partial x_i(\mathbf{p}, I)}{\partial p_j} = \frac{\partial h_i(\mathbf{p}, v(\mathbf{p}, I))}{\partial p_j} - x_j(\mathbf{p}, I)\frac{\partial x_i(\mathbf{p}, I)}{\partial I}$$

Substitution Effect (with a given utility)

Income Effect

Proof

From the identify of the relationship between the Hicksian demand function and the Marshallian demand function, we can have $x_i(\mathbf{p}, e(\mathbf{p}, v)) = h_i(\mathbf{p}, v)$, $e(\mathbf{p}, v) = I$

By substituting the 2nd equation to the 1st equation and then differentiate by p_j, we can get

$$\frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial I}\frac{\partial e}{\partial p_j} = \frac{\partial h_i}{\partial p_j}$$

Furthermore, since

$$\frac{\partial e}{\partial p_j} = h_j(\mathbf{p}, v) = x_j(\mathbf{p}, e(\mathbf{p}, v))$$

we can get

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j} - x_j\frac{\partial x_i}{\partial I}$$

Relationship between Each Function

Utility maximisation ⟷ **Expenditure minimisation**

SLUTSKY Equation

$$\frac{\partial x_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j} - x_j \frac{\partial x_i}{\partial I}$$

Marshallian demand function

$$x_i(\mathbf{p}, I)$$

Roy's identity

$$x_1(\mathbf{p}, I) = \frac{-\partial v(\mathbf{p}, I)/\partial p_i}{\partial v(\mathbf{p}, I)/\partial I}$$

Indirect utility function

$$v(\mathbf{p}, I)$$

Hicksian demand function

$$h_i(\mathbf{p}, \underline{u})$$

$$h_i(\mathbf{p}, \underline{u}) = \frac{\partial e(\mathbf{p}, \underline{u})}{\partial p_i}$$

Expenditure function

$$e(\mathbf{p}, \underline{u})$$

Consumer's Surplus
(消費者余剰)

- Evaluation of Benefit

  ⇨ Cost-benefit analysis (費用便益分析)

- Consumer's Surplus
  - The difference between the maximum price a consumer is willing to pay and the actual price they do pay

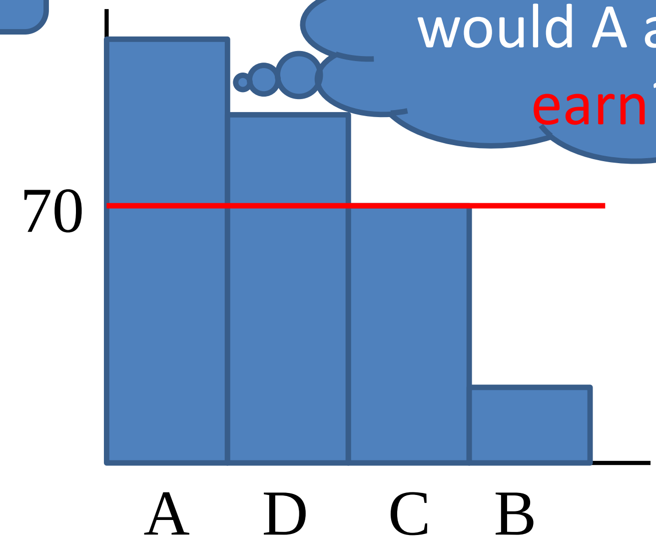
Consumer's Surplus
(消費者余剰)

Suppose you have three computers and your friends are willing to pay following amount of money to get the computer. How much would you charge to your computer?

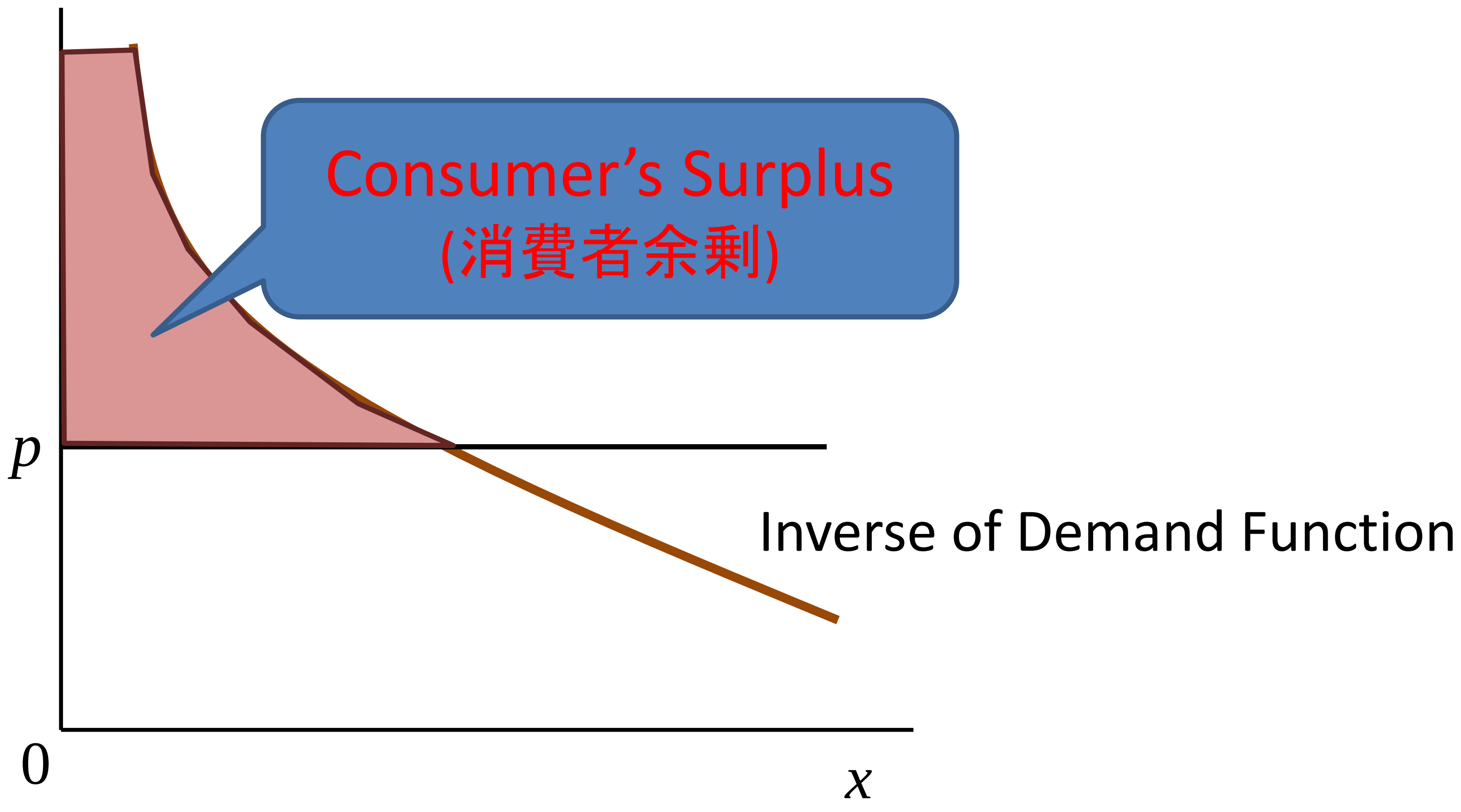
Answer: ¥70 thousands !

Mr. A : ¥130 thousands
Mr. B : ¥20 thousands
Ms. C: ¥70 thousands
Ms. D: ¥100 thousands

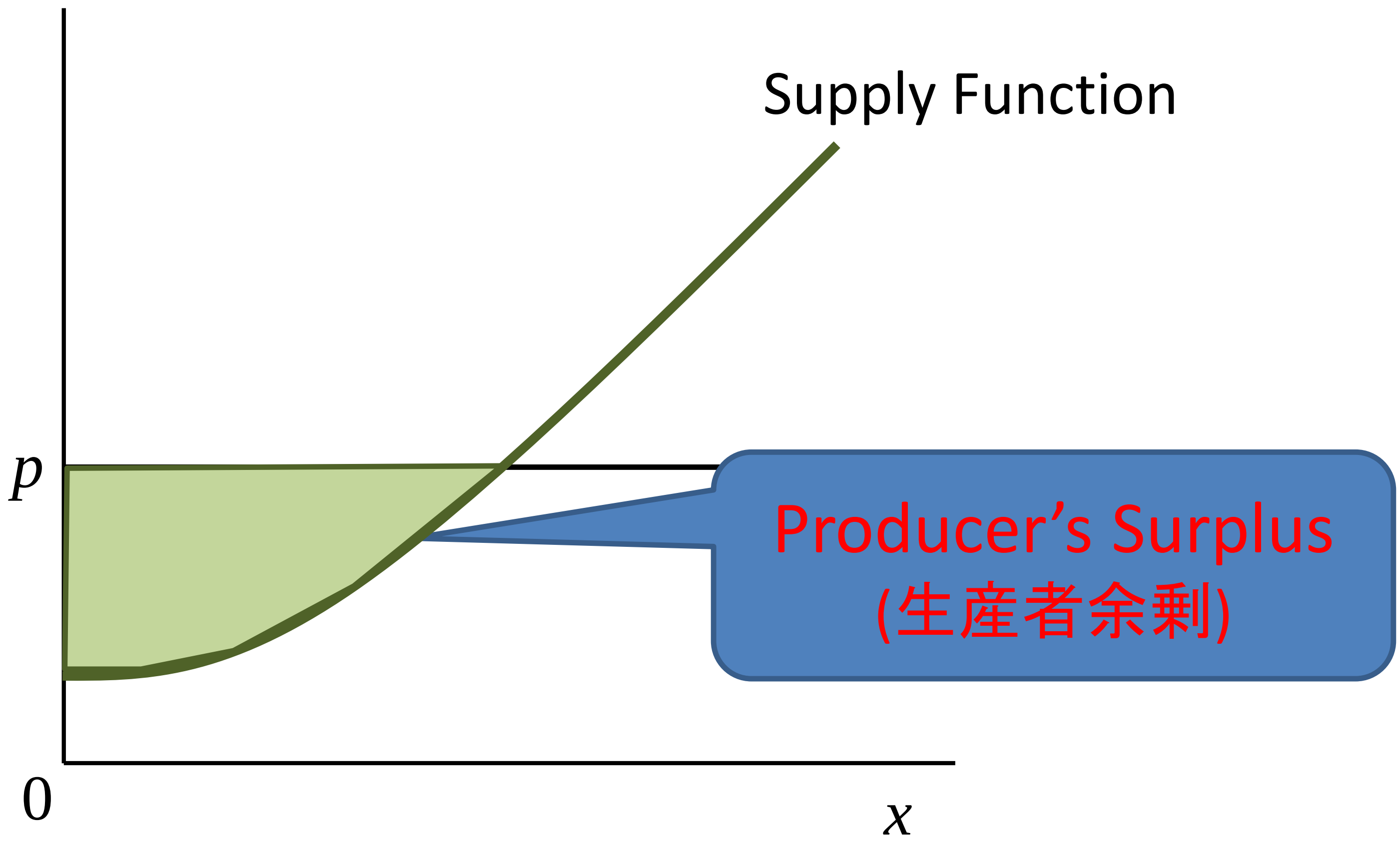
How much would A and D earn?



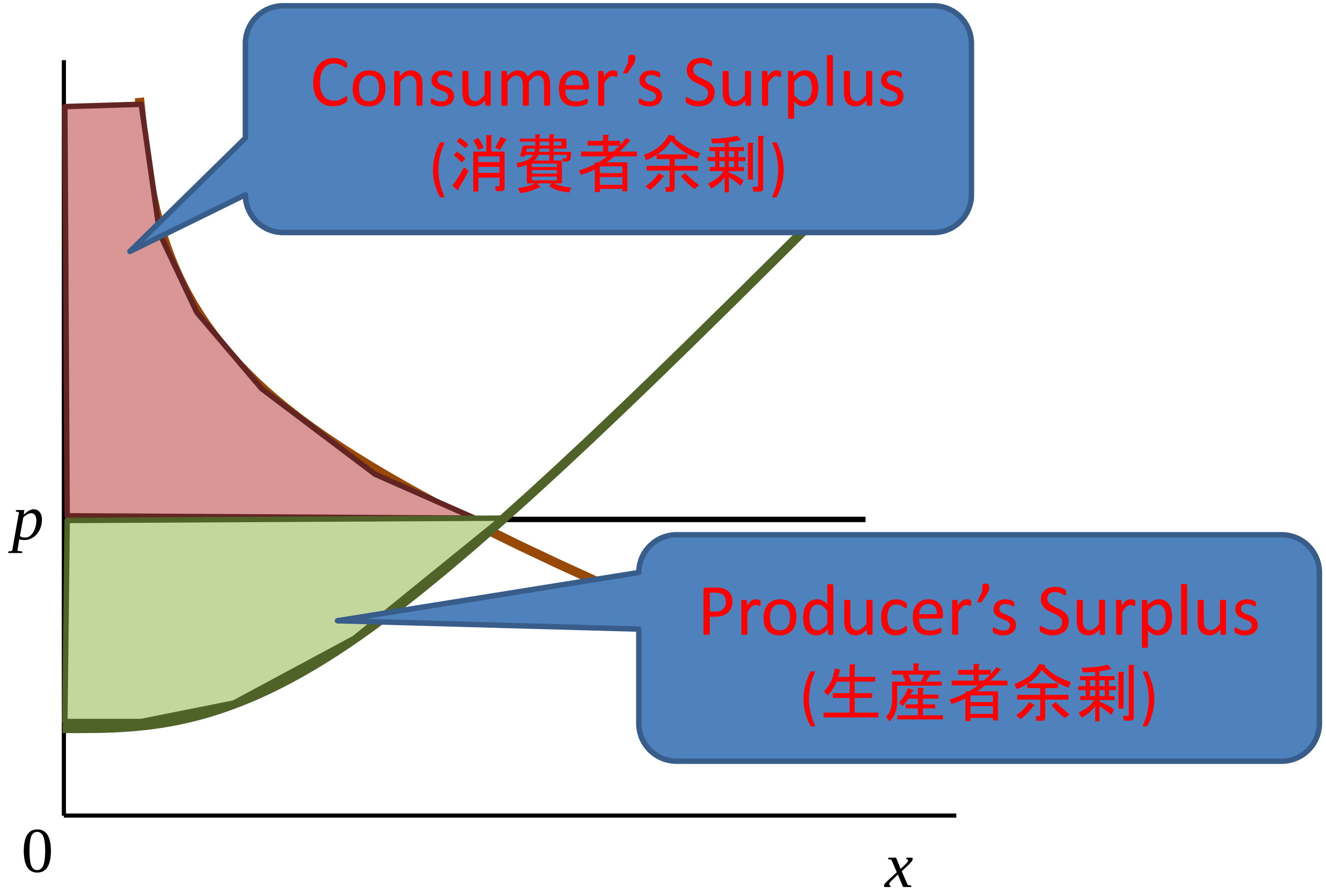
Consumer's Surplus
(消費者余剰)



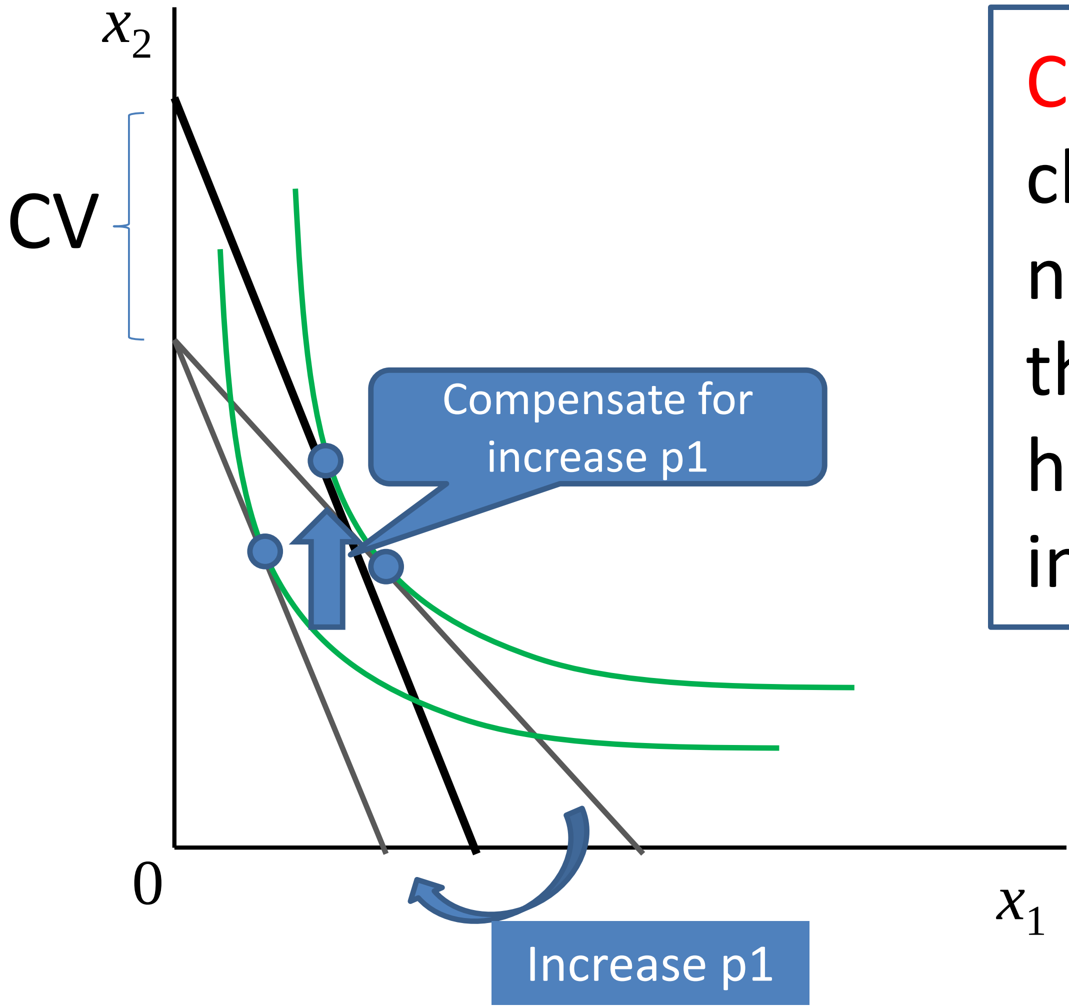
Producer's Surplus
(生産者余剰)



Social Surplus
(社会的余剰)

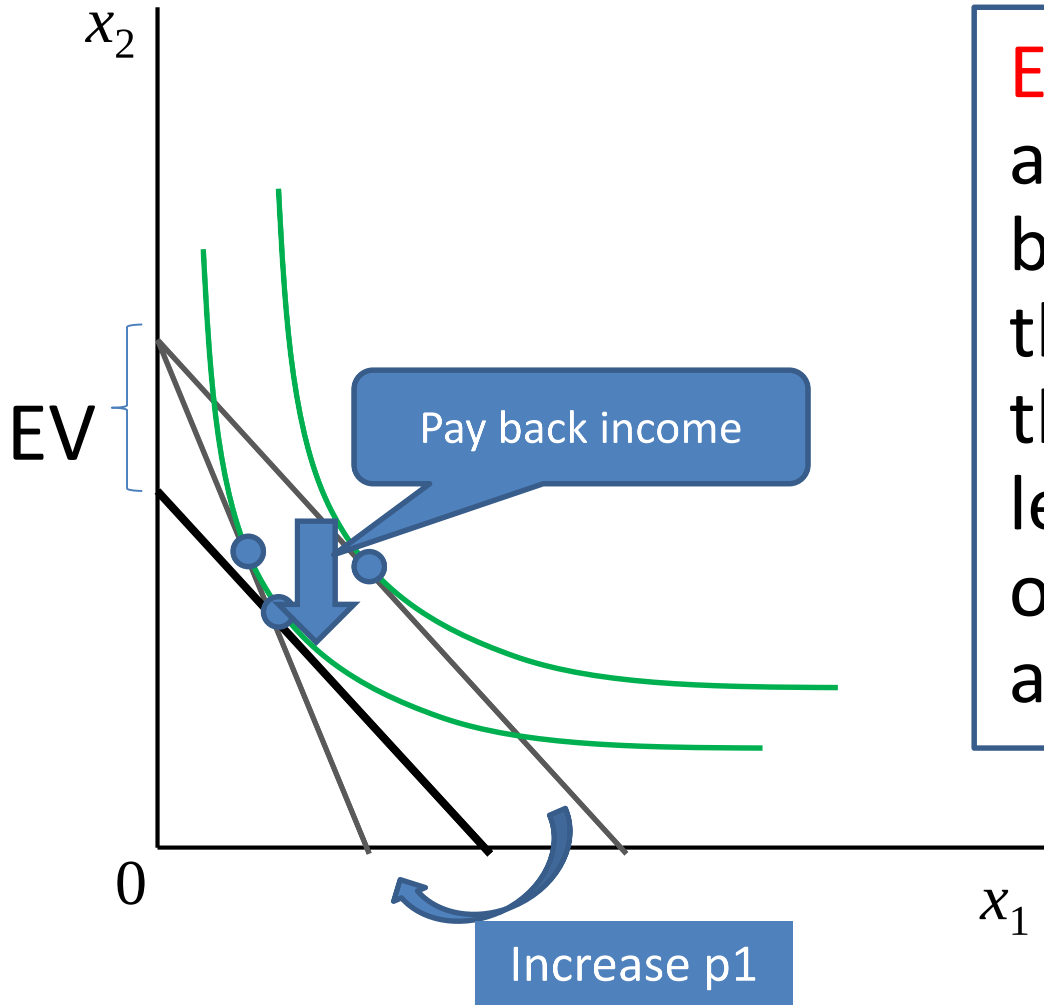


Compensating Variations
(補償変分)



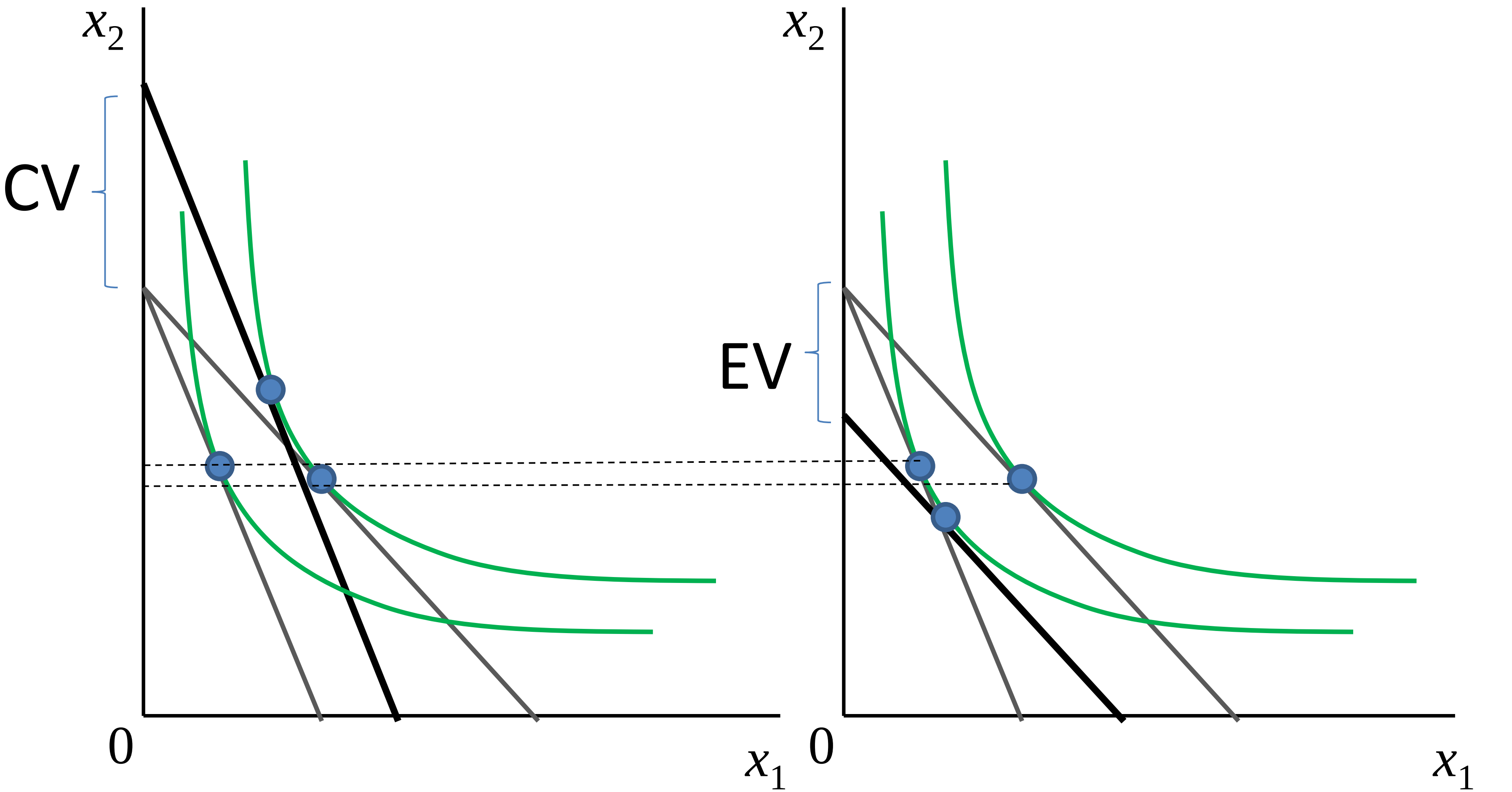
CV represents for the change in income necessary to restore the consumer to his/her original indifference curve

Equivalent Variations
(等価変分)



EV represents for the amount of income to be taken away from the consumer before the price change to leave him/her as well off as (s)he would be after the price change

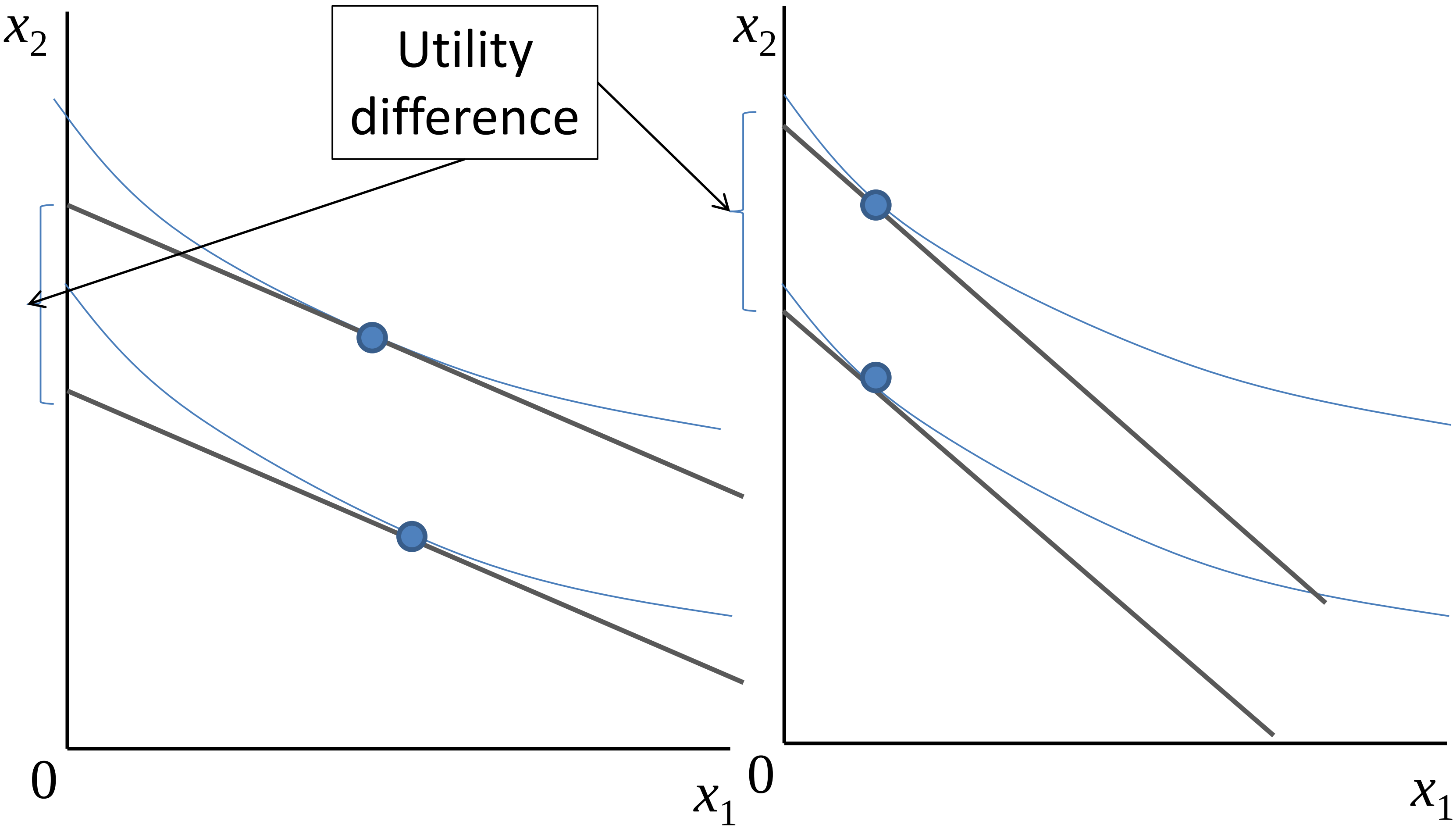
Comparison between CV and EV



Comparison between CV and EV

- Basically, $|CV| \neq |EV|$
  - The amount of money that the consumer would have to pay to compensate him/her for a price change would be different from the amount of money that the consumer would be willing to pay to avoid a price change
- However, $|CV|=|EV|$ in the case of quasilinear utility (準線形効用)
  - where the indifference curves are parallel

In case of Quasilinear Utility



- Utility difference is the same regardless of initial solution