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Kyoto Univ. and UTC Joint Summer Training Course of

Road Infrastructure Asset Management

Bridge Management (4)
Probabilistic Deterioration Prediction

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Probabilistic Deterioration Prediction (1): Markov Chain Model

Markov Transition Probability (1)

Ex）Mr.A moves from area to area, Russia, Tokyo, New York, Hanoi, Melbourne, every month. Which city Mr. A will go depends on which city he stays now.

⇒ Markov Property

The probability distribution of future states of a process depends only upon the current state

Current	Next		
R	R	40%	0.40
	T	30%	0.30
	N	15%	0.15
	H	10%	0.10
	M	5%	0.05

Transpose

R	0.40	0.30	0.15	0.10	0.05
T	0.10	0.50	0.10	0.20	0.10
N	0.20	0.05	0.60	0.05	0.10
H	0.05	0.05	0.30	0.40	0.20
M	0.30	0.10	0.20	0.10	0.30

Markov Matrix Π

Markov Transition Probability (2)

The state being in Russia is defined as 1, Tokyo; 2, New York; 3, Hanoi; 4, Melbourne; 5.

The probability from R to N

= Markov transition probability from state 1 to 3

= π_{13}

$$\Pi = \begin{pmatrix} 0.40 & 0.30 & 0.15 & 0.10 & 0.05 \\ 0.10 & 0.50 & 0.10 & 0.20 & 0.10 \\ 0.20 & 0.05 & 0.60 & 0.05 & 0.10 \\ 0.05 & 0.05 & 0.30 & 0.40 & 0.20 \\ 0.30 & 0.10 & 0.20 & 0.10 & 0.30 \end{pmatrix}$$

- $0 \le \pi_{ij} \le 1$
- $\sum_{j=1}^{J} \pi_{ij} = 1$

Application of Visual Inspection Data to MTPs

5 steps rating system for the results of visual inspection

1; new construction, 5; limit in service

Different Point from the previous example

⇒ Rating can not transit to better condition state.
Condition of bridges can not be recovered as long as no repair/rehabilitation carried out.

In the previous example, it is equivalent to setting up a restriction which Mr. A is not able to move from the south to the north.

$$\Pi = \begin{pmatrix} 0.40 & 0.30 & 0.15 & 0.10 & 0.05 \\ 0.10 & 0.50 & 0.10 & 0.20 & 0.10 \\ 0.20 & 0.05 & 0.60 & 0.05 & 0.10 \\ 0.05 & 0.05 & 0.30 & 0.40 & 0.20 \\ 0.30 & 0.10 & 0.20 & 0.10 & 0.30 \end{pmatrix}$$

→ $\pi_{ij} = 0 \ (i > j)$

Significant Problem of Application

Existing Simple Method

No. of Samples (Visual Inspection Data)

state 1 to1; 50 samples, 1 to 2; 30, 1 to 3; 15, 1 to 4; 4, 1 to 5; 1

Markov transition probability (relative frequency)

π_{11}=0.50, π_{12}=0.30, π_{13}=0.15, π_{14}=0.04, π_{15}=0.01.

$$\Pi = \begin{pmatrix} 0.50 & 0.30 & 0.15 & 0.04 & 0.01 \\ 0 & 0.60 & 0.30 & 0.05 & 0.05 \\ 0 & 0 & 0.70 & 0.20 & 0.10 \\ 0 & 0 & 0 & 0.75 & 0.25 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The existing method requires uniformity of the sampling interval.

⟷ **Visual inspection intervals are not uniformity.**

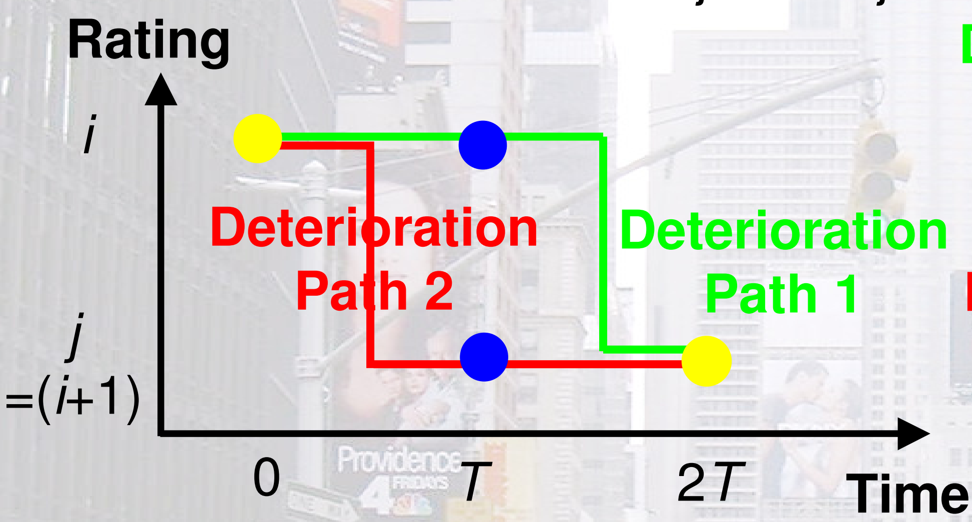
Neglect nonuniformity or extract only the data in same sampling intervals

Concept of Proposed Method

Visual Inspection Interval: 2*T* years, Rating: *i to j*（*i*+1）

Markov Transition Probability: π_{ij}^{2T}

How should we represent π_{ij}^{2T} **by** π_{ij}^{T} **with inspection interval *T* year?**



Deterioration Path 1:

$h(0)=i \rightarrow h(T)=i \rightarrow h(2T)=j$

$\underbrace{h(0)=i \rightarrow h(T)=i}_{\pi_{ii}^{T}}$, $\underbrace{h(T)=i \rightarrow h(2T)=j}_{\pi_{ij}^{T}}$

Deterioration Path 2:

$h(0)=i \rightarrow h(T)=j \rightarrow h(2T)=j$

$\underbrace{h(0)=i \rightarrow h(T)=j}_{\pi_{ij}^{T}}$, $\underbrace{h(T)=j \rightarrow h(2T)=j}_{\pi_{jj}^{T}}$

$$\pi_{ij}^{2T} = (\pi_{ii}^{T}\pi_{ij}^{T} + \pi_{ij}^{T}\pi_{jj}^{T}) = \sum_{k=i}^{j}\pi_{ik}^{T}\pi_{kj}^{T}$$

General Case

$$\pi_{ij}^{nT} = \sum_{k_1=i}^{j}\pi_{ik_1}^{T}\sum_{k_2=k_1}^{j}\pi_{k_1k_2}^{T}\cdots \cdots\sum_{k_{n-1}=k_{n-2}}^{j}\pi_{k_{n-2}k_{n-1}}^{T}\pi_{k_{n-1}j}^{T}$$

Estimation of MTP by Maximum-Likelihood method

Likelihood Function based on Simultaneous occurrence probability of Visual Inspection Results

$$L = \prod_{n=1}^{N} \prod_{i=1}^{J} \prod_{j-1}^{J} (\pi_{ij}^{nT})^{m_{ij}^{n}}$$

Log Likelihood Function

$$\ln L = \sum_{n=1}^{N} \sum_{i=1}^{J} \sum_{j=1}^{J} m_{ij}^{n} \cdot \pi_{ij}^{nT}$$

In order to decide the unknown parameters which maximize the likelihood function, the log likelihood function is differentiate partially by each unknown parameter, the obtained nonlinear simultaneous equation is solved by numerical calculation

$$\frac{\partial \ln L}{\partial \pi_{ij}^{T}} = 0$$

Empirical Verification

Application for Actual Visual Inspection Data of RC Decks in NYC
7-Level Rating Standards (RC Decks)

Ratings	Physical Meanings
1	Deck is new or near new, almost no sign of deterioration
2	Between 1 & 3
3	Only localized areas of leakage
4	Between 3 & %
5	75% or more of the deck has leakage. Only localized spalled areas. Efflorescence along the girder top flanges
6	Between 5 & 7
7	Heavy spalling, Heavy efflorescence, Punch through has occurred or is likely, Deck saturated to point that concrete is rubble.

No. of Samples for Each Inspection Interval

Inspection Intervals (Years)	No. of Samples
1	14,030
2	18,312
3	479
Total	32,821

Estimation Results of MTPs

Rating	1	2	3	4	5	6	7
1	0.713	0.281	0.006	0	0	0	0
2	0	0.793	0.189	0.018	0	0	0
3	0	0	0.852	0.142	0.006	0	0
4	0	0	0	0.893	0.101	0.006	0
5	0	0	0	0	0.917	0.063	0.020
6	0	0	0	0	0	0.919	0.081
7	0	0	0	0	0	0	1

- **Employing all data including inspection interval 1 to 3 years.**
- **The decrease in the rating during a single interval is limited to 2 levels.**
- **When $i > j$, $\pi_{ij}=0$. $\pi_{JJ}=1$ means absorbing condition.**
- **For every rating, the diagonal part (deterioration does not progress and the rating remains constant)shows the maximum of the MTPs.**
- **As deterioration progress, the speed of the deterioration becomes slower**

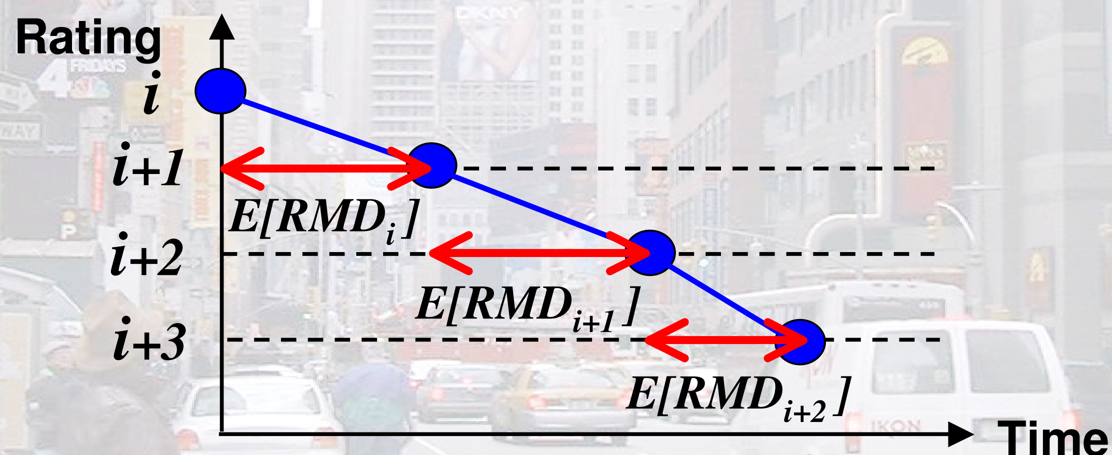
Expected Deterioration Path

The expected lifetime of the rating to transit from *i* to *i*+1:

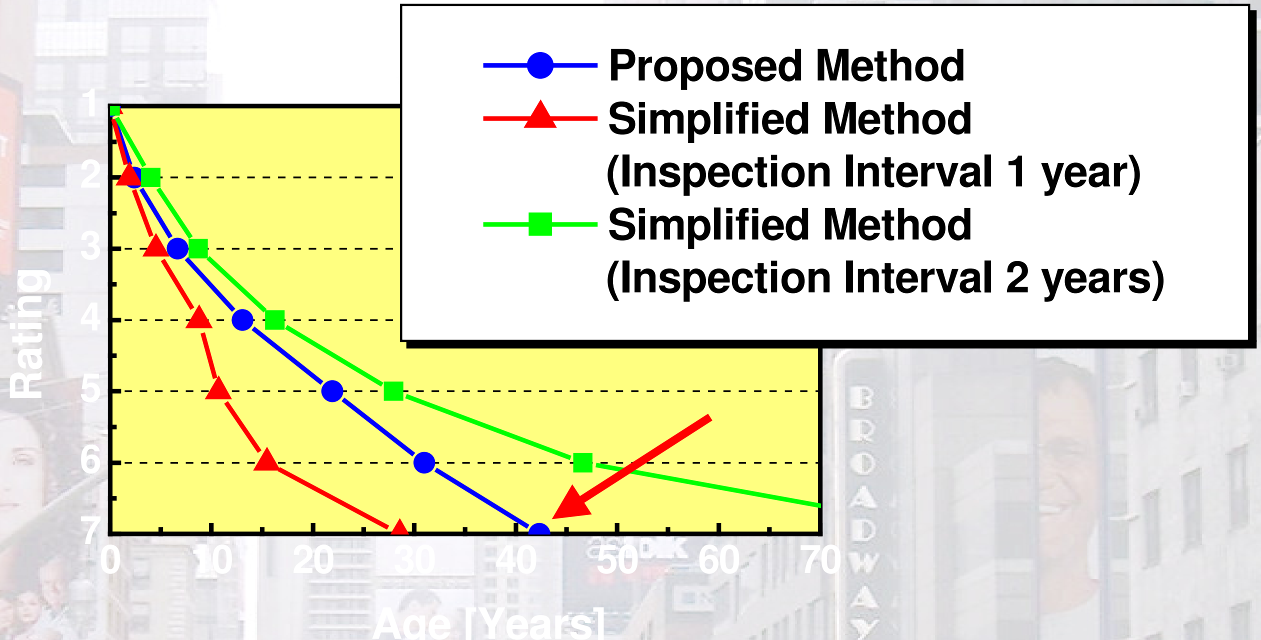
$$E[RMD_i] = T \cdot \pi_{iJ}^{T} + \sum_{\ell=1}^{\infty} \sum_{m=i}^{J-1} (\ell T + T) \cdot \pi_{im}^{\ell T} \pi_{mJ}^{T} - \sum_{m=i+1}^{J-1} E[RMD_m]$$

The 1st + 2nd term: The expected No. of years between *i* and *J-1*.

The 3rd term: the sum of the expected No. of years between i+1 to *J*-1.



Expected Path for RC Decks



● **The simplified method does not satisfy the time adjustment condition.**

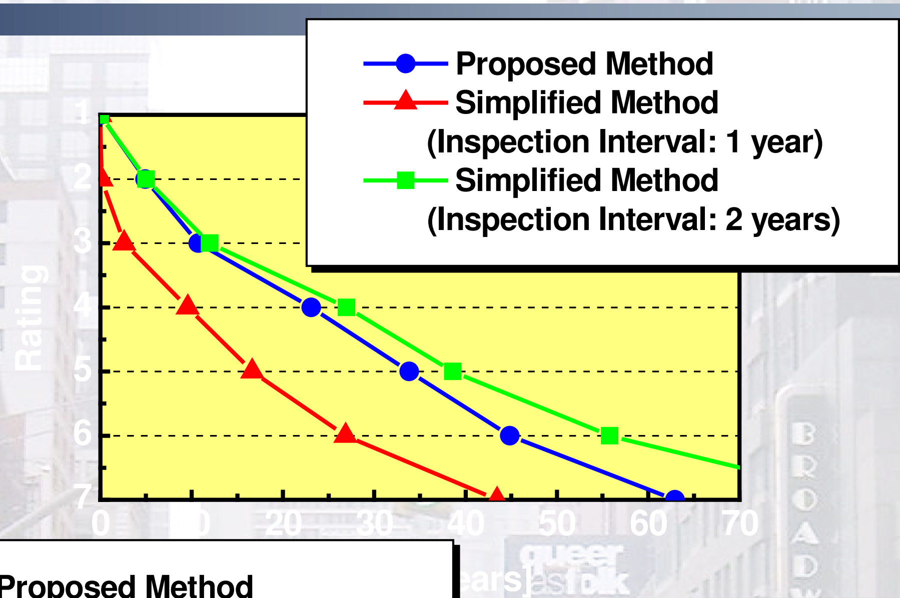
$$\{\Pi^{(1)}\}^2 \neq \Pi^{(2)}$$

The interval of visual inspection is not determined randomly, but determined by considering physical properties of the RC decks.

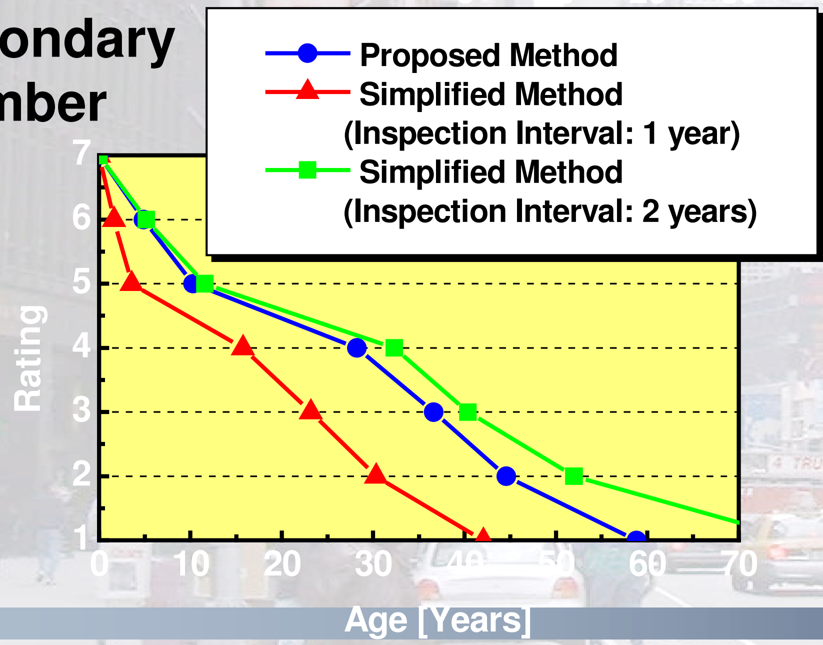
● **The curve of the proposed method is between those obtained by the simple aggregation of the one-year and two-year inspection intervals**

Expected Path for Another Member

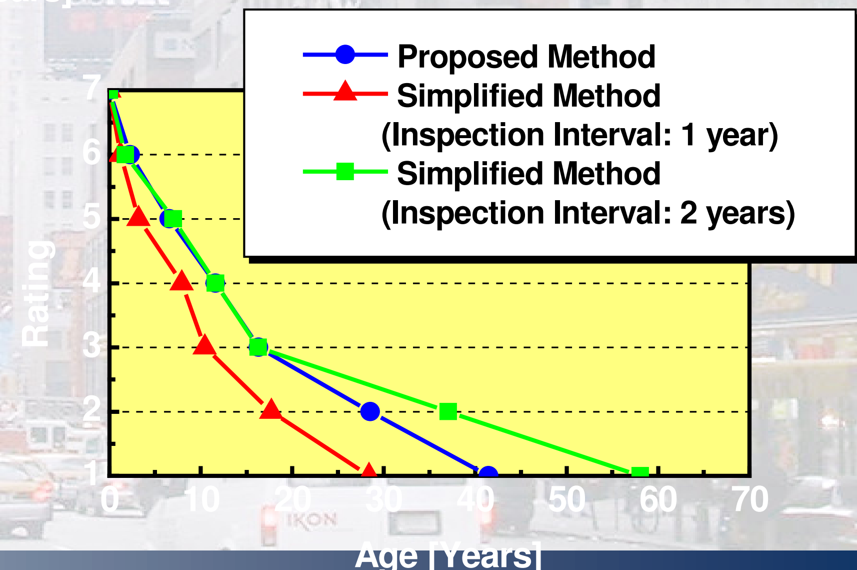
Primary Member



Secondary Member



Joint



Calculation of Transition of The Rating Distribution

○**Definition of the state vector $\boldsymbol{x}_t$:**

Relative frequencies of each rating at a given time point *t*

$$x_t = [x_1(t) \quad x_2(t) \quad \cdots \quad x_J(t)]$$

$x_1(t)$ → $\dfrac{\text{The No. of samples of rating } i \text{ at time point } t}{\text{Total no. of samples}}$

○**The state vector at time point *t*+1**

$$x_{t+1} = x_t \Pi$$

○**The state vector at an arbitrary time point ?**

The state vector at time point *t*+2:

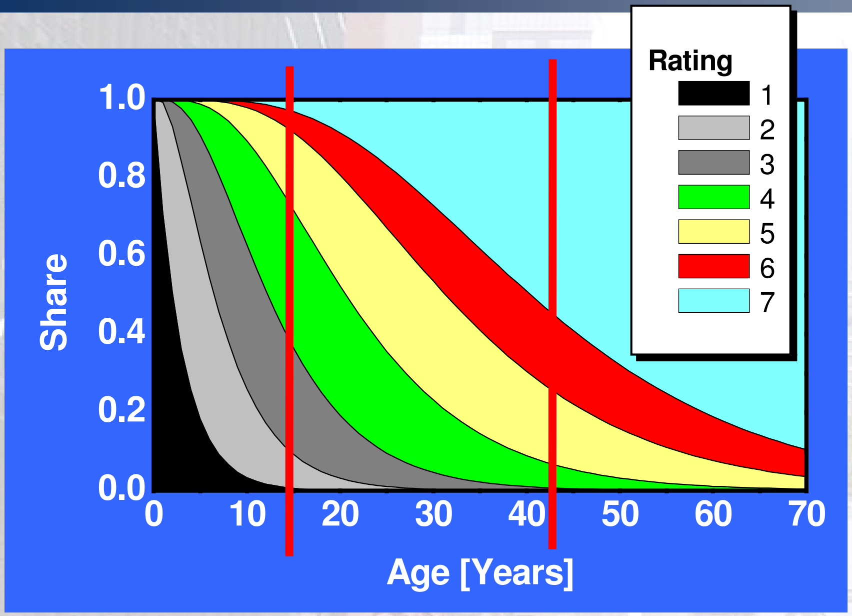
conducting recalculation after substituting $\boldsymbol{x}_{t+1}$ with $\boldsymbol{x}_t$

At arbitrary time point *t*+*a*:

repeating the above calculation the necessary no. of times

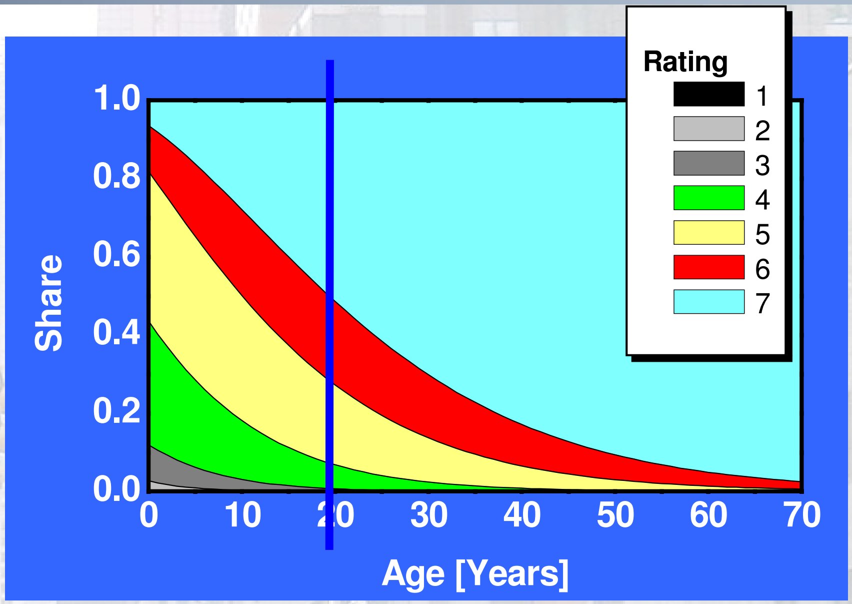
a

Transition of Rating Distribution



a) All RC decks are the state of new construction.

Initial Vector; x =[1 0 0 0 0 0 0]



b) Actual visual inspection results of NYC in 1995

Initial Vector; x =[0.001 0.026 0.092 0.317 0.382 0.117 0.0065]

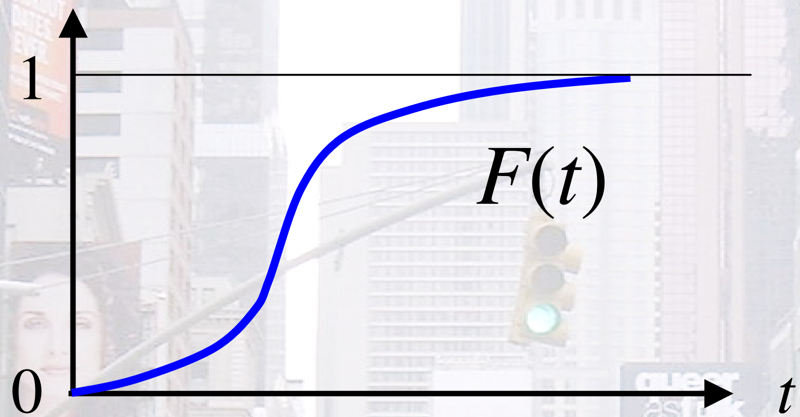
- 15 years later, very few RC decks still keep rating 1.
- At around 42 years, 50% of them have reached the limit of use(rating 7).
- Under inappropriate management, in 20 years the ratio of the rating 7 will amount to 50%.

Probabilistic Deterioration Prediction (2):

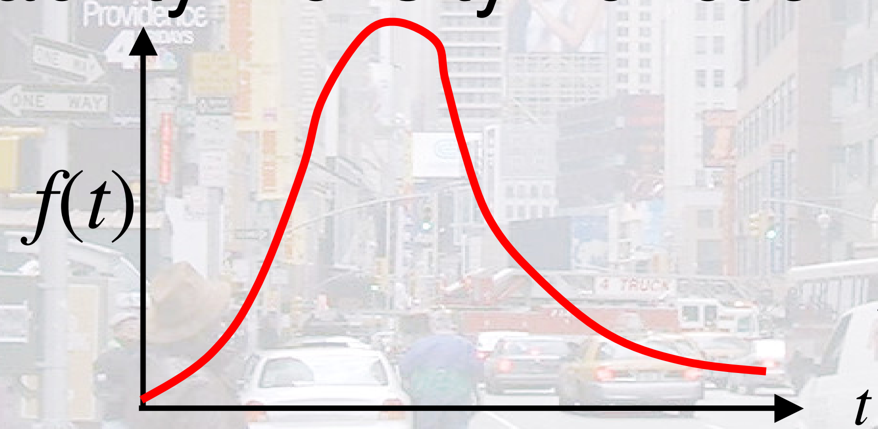
Random Proportional Weibul Hazard Deterioration Model

Statistical Deterioration Prediction by Hazard Model

- Failure Cumulative Distribution Function: $F(t)$



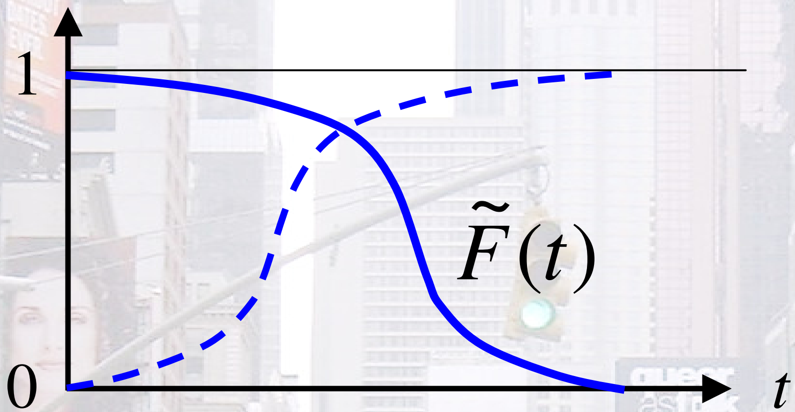
- Probability Density Function: $f(t)$



$$F(t) = \int_0^t f(u)du$$

Statistical Deterioration Prediction by Hazard Model

- Survival Distribution Function: $\tilde{F}(t)=1- F(t)$



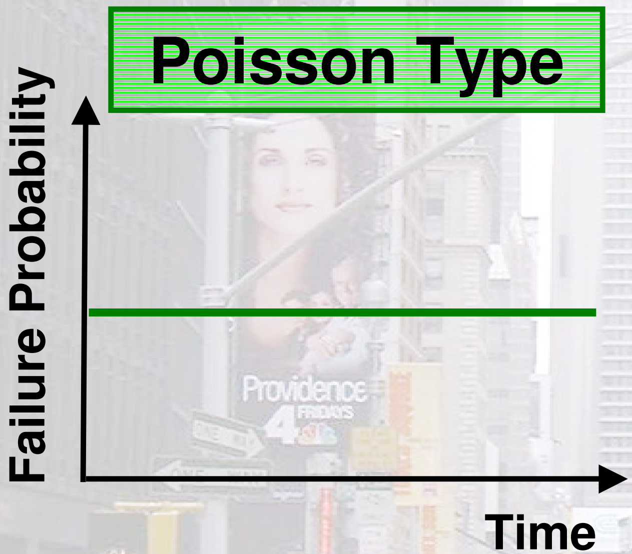
- Conditional Probability(the equipment survives until time *t*, and moreover fails during the time interval period $t,t+\Delta t$)

$$\lambda(t)\Delta t = \frac{f(t)\Delta t}{\tilde{F}(t)}$$

Statistical Prediction by Hazard Model

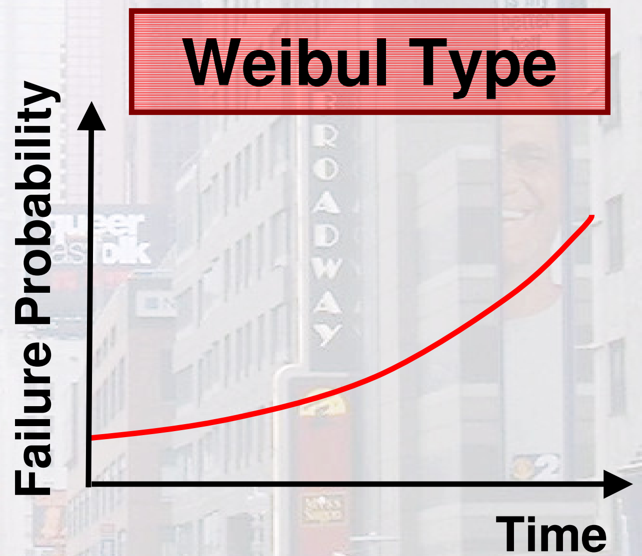
Two-Type Deterioration Pattern

Time Dependent or not of Failure Probability



Exponential

Hazard Function



Weibul

Hazard Function

Weibul Deterioration Hazard Model

$$ï(t) = ç\, m\, t^{m\,A\,1}$$

$$f(t) = ç m t^{m A 1} \exp(Äç t^{m})$$

$$\bar{F}(t) = \exp(Äç t^{m})$$

Disadvantage in the Existing Hazard Model

Deterministic Hazard Rate

- Same Deterioration Process under the Same Condition

How we can model the heterogeneity of Individual equipments?

Random Proportional Weibul Hazard Model

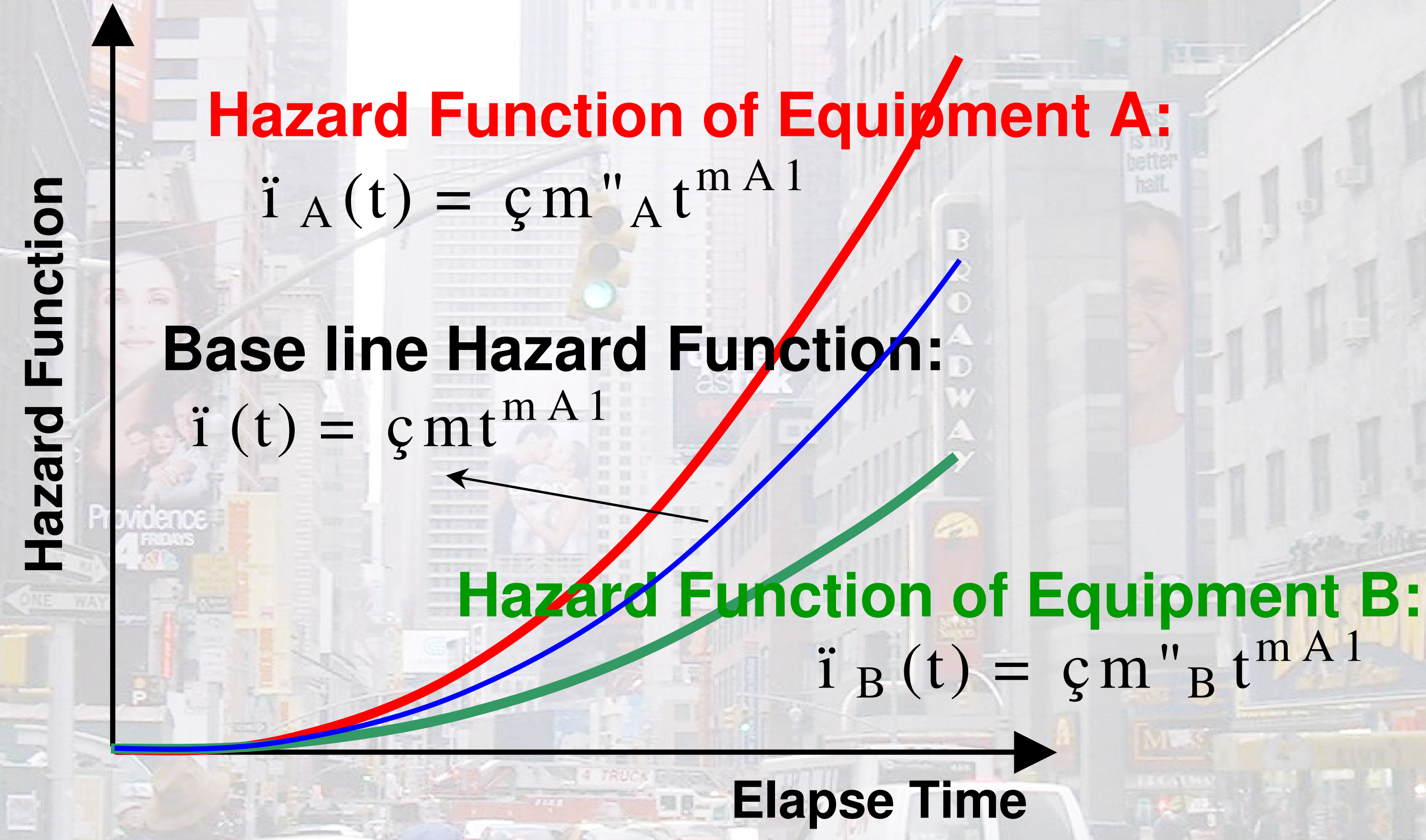
$$ï_{ij}(t^{k}_{ij}) = "_{ij}\,ç_{i}\,m\,(t^{k}_{ij})^{m\,Ä\,1}$$

Heterogeneity Parameter that is subject to a certain probability Distribution

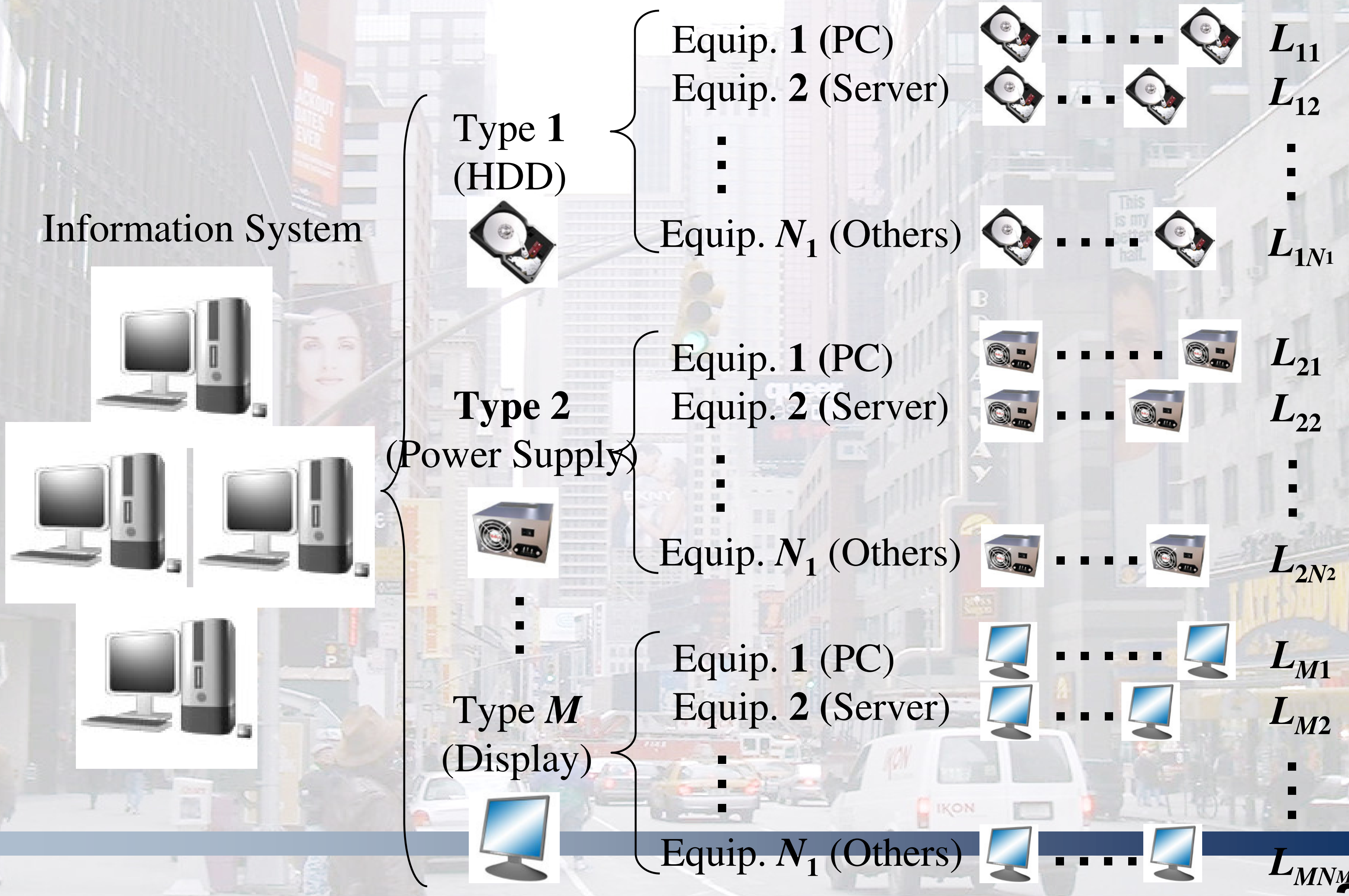
$$f_{ij}(t^{k}_{ij}) = "_{ij}\,ç_{i}\,m(t^{k}_{ij})^{m\,Ä\,1}\,\mathrm{expf}\,Ä\,"_{ij}\,ç_{i}(t^{k}_{ij})^{m}\,g$$

$$\tilde{F}_{ij}(t^{k}_{ij}) = \mathrm{expf}\,Ä\,"_{ij}\,ç_{i}(t^{k}_{ij})^{m}\,g$$

Concept of Random Proportional Hazard Model



Information Infrastructures



Considerable Heterogeneity of Samples

- HDD
- Power Supply
- Data Processing Device

} → 3 Types

↓

Installed in 9 Sub-System

↓

3 Usage; PC, Server, Others

- Power Supply has Only one Usage

Number of Samples

		HDD	Power	Proc.
Sub Sys. 1	PC	1	5	1
	Server	1	—	1
	Others	—	—	2
Sub Sys. 2	PC	9	96	10
	Server	17	—	7
	Others	1	—	3
Sub Sys. 3	PC	3	81	3
	Server	23	—	15
	Others	2	—	15
Sub Sys. 4	PC	12	27	5
	Server	22	—	7
	Others	16	—	16
Sub Sys. 5	PC	5	12	4
	Server	15	—	9
	Others	—	—	—

		HDD	Power	Proc.
Sub Sys. 6	PC	4	17	4
	Server	8	—	6
	Others	—	—	2
Sub Sys. 7	PC	2	7	2
	Server	9	—	2
	Others	—	—	4
Sub Sys. 8	PC	32	51	23
	Server	13	—	7
	Others	—	—	16
Sub Sys. 9	PC	4	10	3
	Server	3	—	3
	Others	—	—	5

Many kinds of Little samples

Available Information (Failure History)

Failure History of All Equipments

$$\tilde{N} = (\grave{o}_1 ; \text{ÅÅÅ}; \grave{o}_M)$$

Failure History of Equipment Type *i*

$$\grave{o}_i = (\grave{o}_{i1} ; \text{ÅÅÅ}; \grave{o}_{iN_i})$$

Failure History of Device *j* of Type *i*

$$\grave{o}_{ij} = (\acute{e}^1_{ij} ; t^1_{ij}); \text{ÅÅÅ}; (\acute{e}^{L_{ij}}_{ij} ; t^{L_{ij}}_{ij})$$

$$(i = 1; \text{ÅÅÅ}; M ; j = 1; \text{ÅÅÅ}; N_i)$$

$\acute{e}^k_{ij}$ **: Dummy Variable**

$\acute{e}^j_i = 0$ **: Failure**

$\acute{e}^j_i = 1$ **: Otherwise**

t^k_{ij} **: Failure Time or Elapse Time**

Estimation of Hazard Model

$$\lambda_{ij}(t_{ij}^{k}) = \underline{\varepsilon_{ij}}\,\gamma_i\, m\,(t_{ij}^{k})^{m-1}$$

Standard Gamma Distribution

$$\bar{g}(\varepsilon_{ij} : \alpha) = \frac{\alpha^{\alpha}}{\Gamma(\alpha)}\varepsilon_{ij}^{\alpha-1}\exp(-\alpha\varepsilon_{ij})$$

Objectives: HDD, Power Supply, Data Processing Device

No. of Samples: Total 693

Available Data：Failure History (1998-2006.9)

2-Step Estimation Method

1. Estimation of $ç_i$ m $û$ by Maximum Likelihood Method

$$\ln L(Ñ;í) = \sum_{i=1}^{M}\sum_{j=1}^{N_i} \ln L_{ij}(ò_{ij} : í_i)$$

$$= N û \ln û - \sum_{i=1}^{M}\sum_{j=1}^{N_i} (s_{ij} + û)\ln(û + ç_i ú_{ij})$$

$$+ \sum_{i=1}^{M}\sum_{j=1}^{N_i}\sum_{k=0}^{s_{ij}-1} \ln(û + k) + \sum_{i=1}^{M}\sum_{j=1}^{N_i}\sum_{k=1}^{t_{ij}} é_{ij}^{k} \left\{ \ln ç_i + \ln m + (m - 1)\ln t_{ij}^{k} \right\}$$

2. Calculation of Heterogeneity Parameter $¨_{ij}$

$$\hat{'}_{ij}(\hat{í}_i) = \frac{s_{ij} + \hat{û} - 1}{\hat{û} + \hat{ç}\hat{ú}_{ij}}$$

Estimation Results

$$\text{ï}_{ij}(t^{k}_{ij}) = \text{"}_{ij}\,\text{ç}_{i}\,m(t^{k}_{ij})^{m\,\text{Ä}\,1}$$

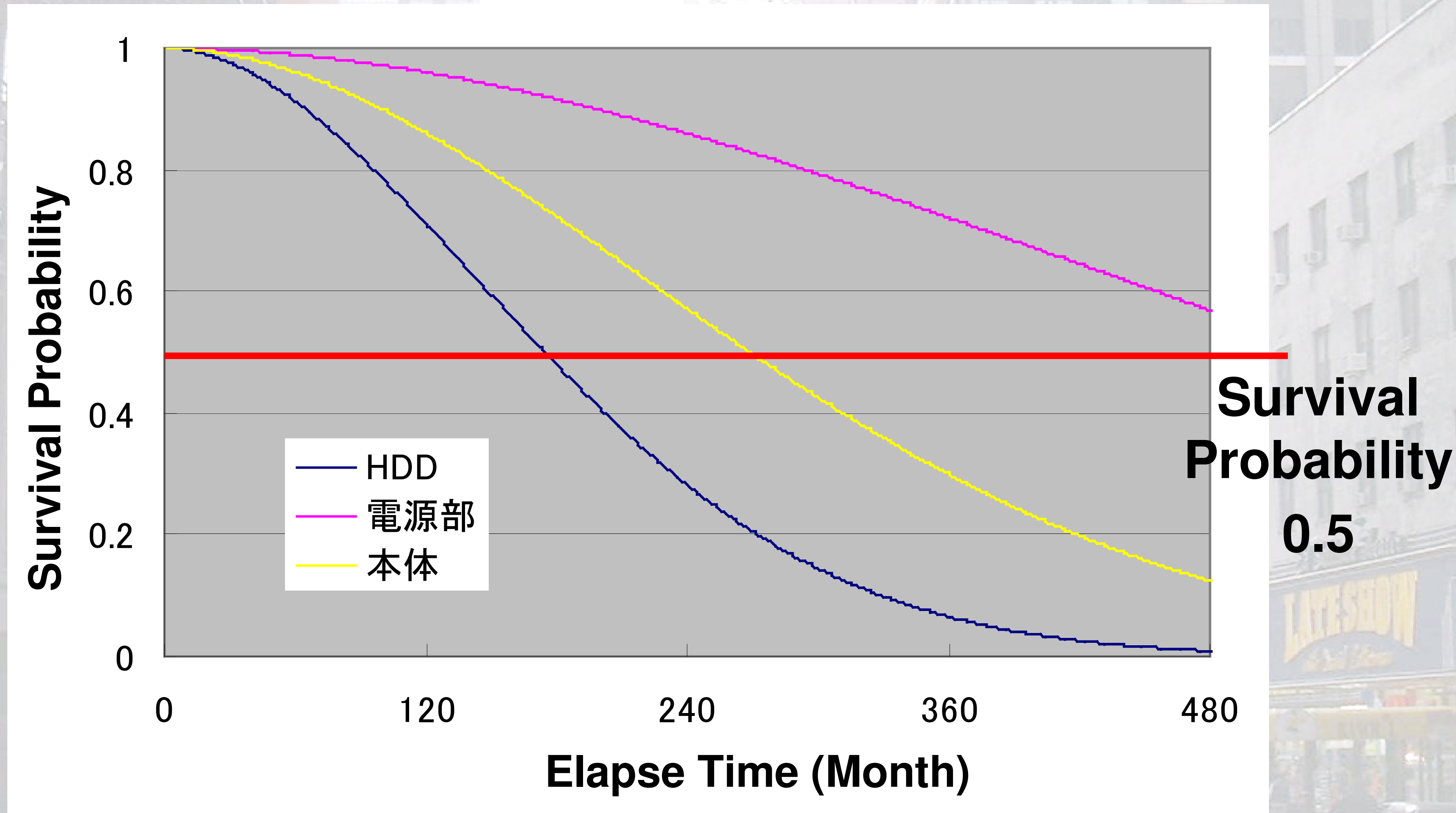
	γ			m	ϕ
	γ_1	γ_2	γ_3		
Estimator	**1.251E-5**	**1.631E-6**	**5.293E-6**	**2.174**	**1.193**
***t*-value**	**-5.104E6**	**-2.311E7**	**-9.182E6**	**49.031**	**2.182**
Log-Likelihood	**-402.441**				

Calculation Results of Heterogeneity Parameters

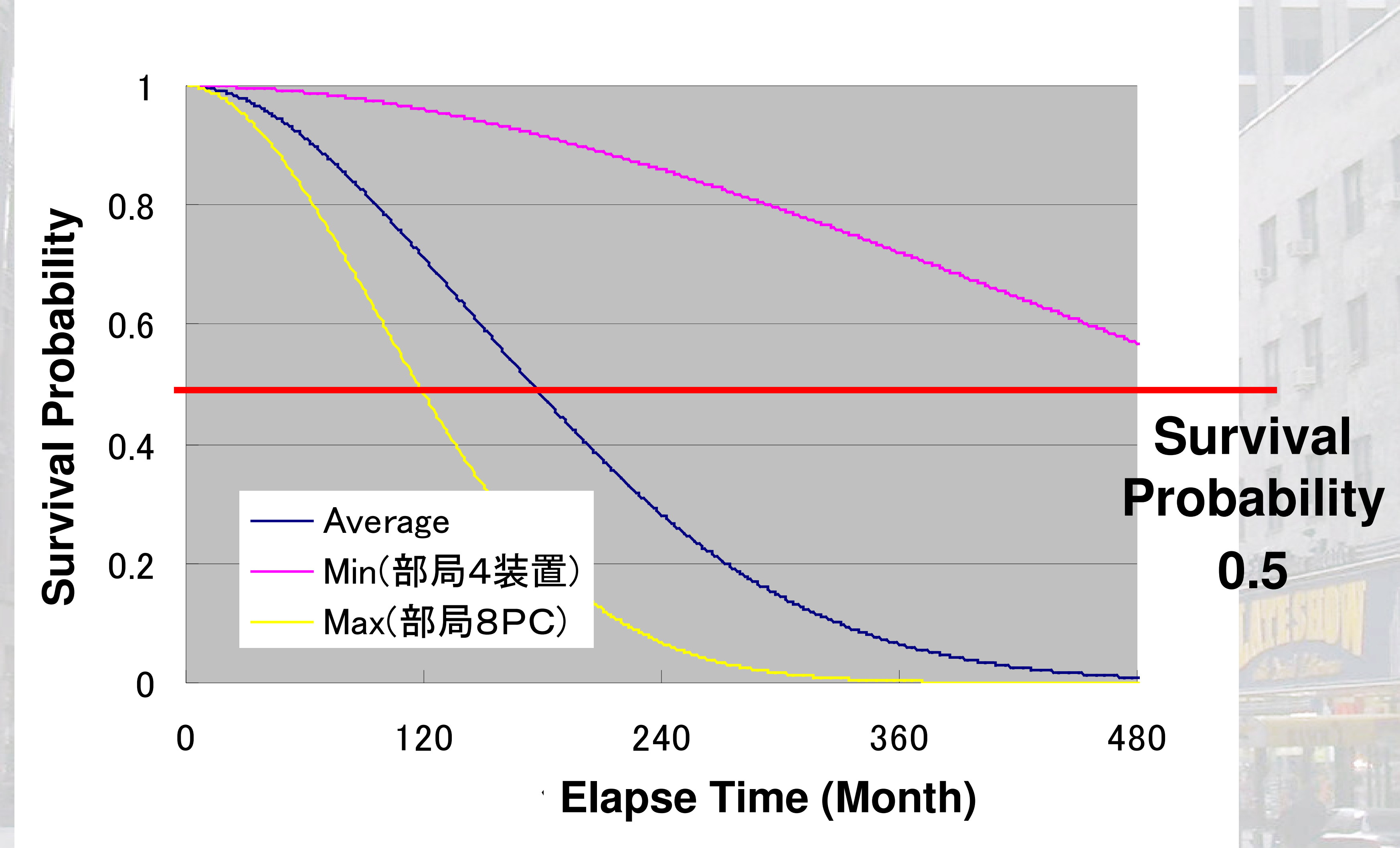
		HDD	Power	Proc.
Sub Sys. 1	PC	0.474	0.253	0.473
	Server	0.462	—	0.462
	Others	—	—	0.306
Sub Sys. 2	PC	0.392	0.171	0.814
	Server	1.76	—	0.405
	Others	0.488	—	0.301
Sub Sys. 3	PC	0.454	0.114	0.454
	Server	0.688	—	0.276
	Others	0.430	—	0.879
Sub Sys. 4	PC	1.28	0.170	0.772
	Server	0.780	—	0.756
	Others	0.220	—	0.837
Sub Sys. 5	PC	0.818	0.287	0.386
	Server	0.679	—	0.715
	Others	—	—	—

		HDD	Power	Proc.
Sub Sys. 6	PC	0.427	0.282	0.423
	Server	0.377	—	0.849
	Others	—	—	0.447
Sub Sys. 7	PC	0.457	0.278	0.457
	Server	1.21	—	0.436
	Others	—	—	0.326
Sub Sys. 8	PC	3.92	1.15	0.557
	Server	1.50	—	1.30
	Others	—	—	0.652
Sub Sys. 9	PC	0.883	0.707	0.583
	Server	0.437	—	0.961
	Others	—	—	0.402

Survival Curve of Each Type



Survival Curve Considering Heterogeneity (HDD)



Thank you for your attention.

Difference Between Civil and Information Systems

- 故障発生から直接的損失発生までタイムラグの短さ
- 複雑な階層構造

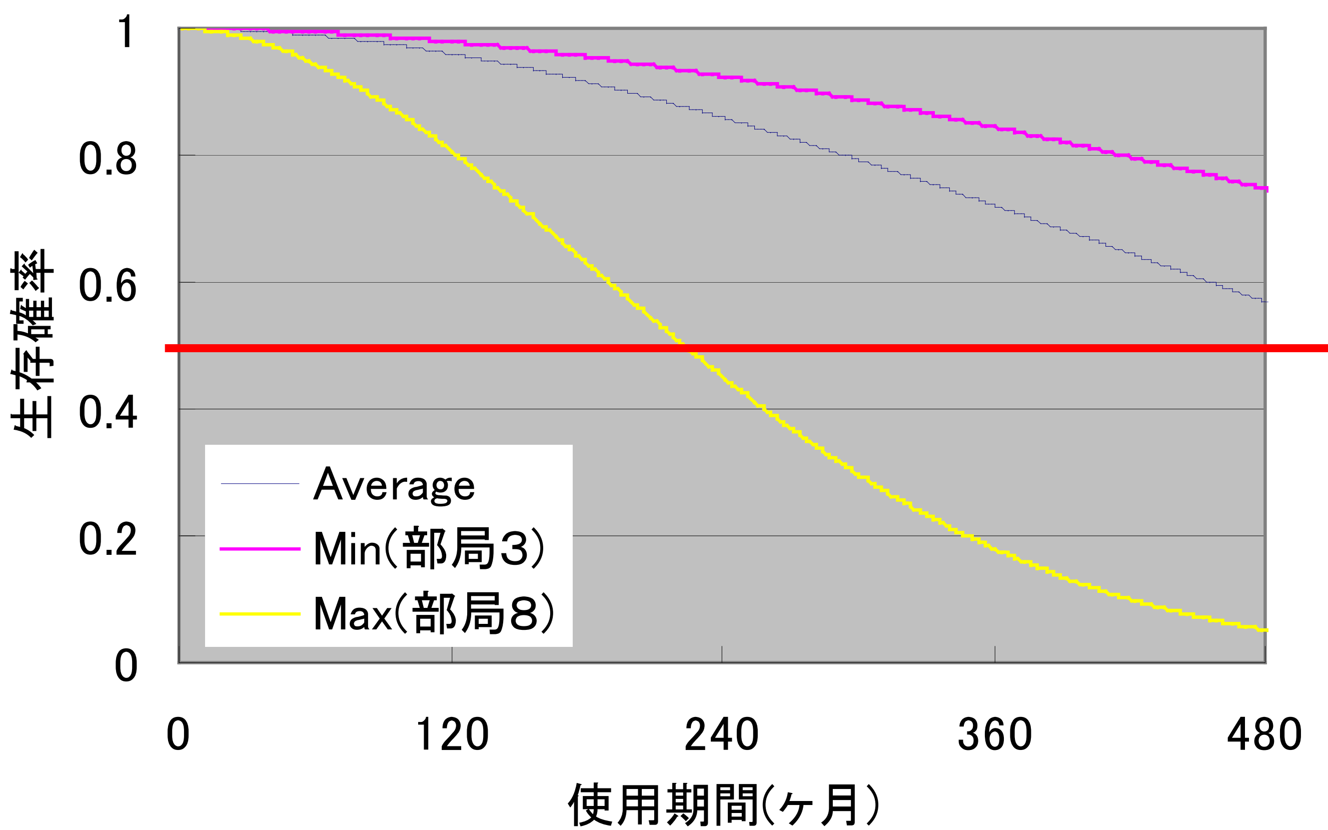
    下位の階層での故障が上位の機能を停止させる可能性

- 故障の横断的波及効果（情報伝達障害）
- 個別システムの高速な機能・性能の陳腐化

    2,3年周期の技術革新サイクル

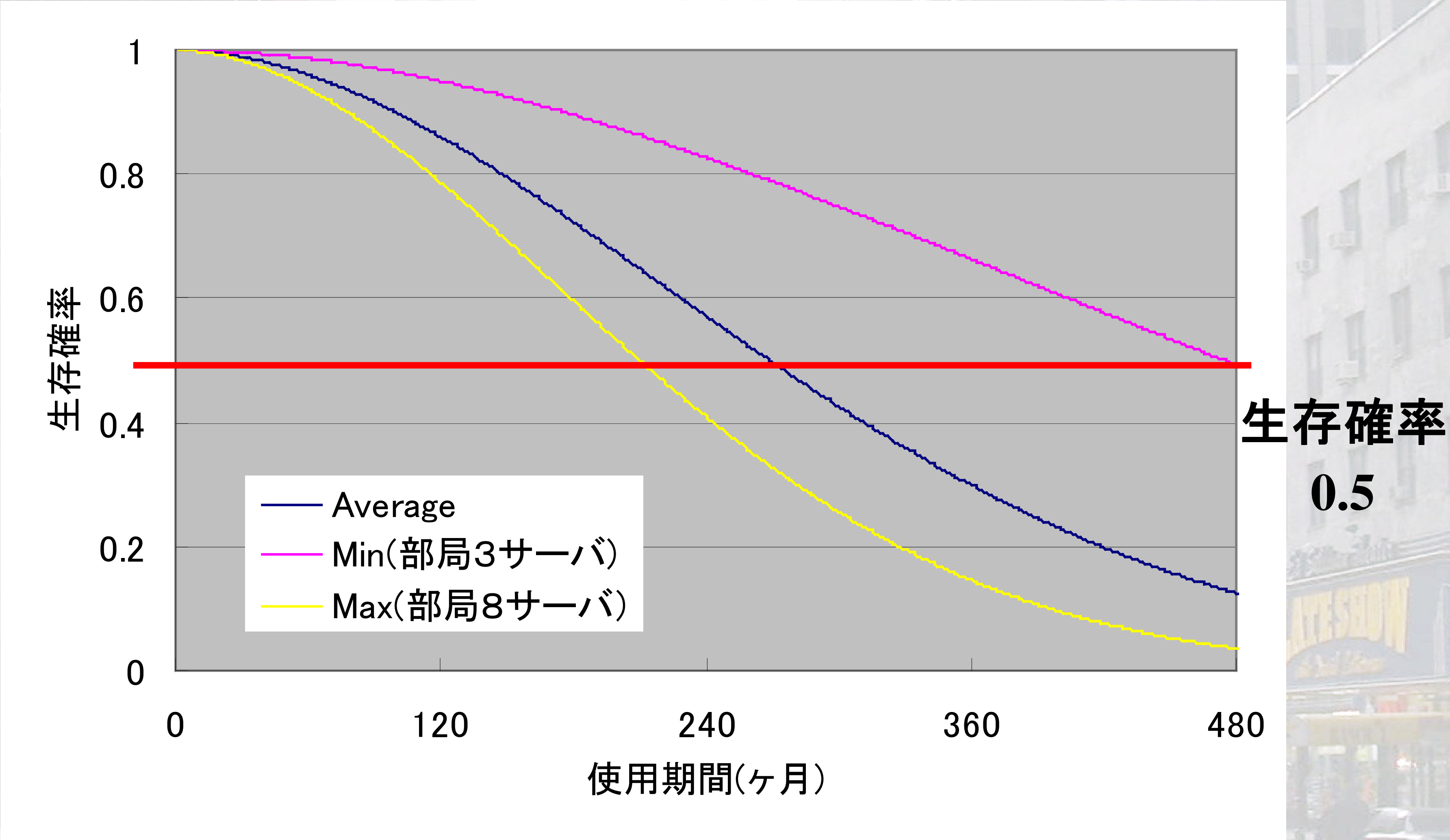
- 豊富な代替案の存在
- 顧客サービス水準の高度化

異質性を考慮した生存曲線(電源部)



生存確率
0.5

異質性を考慮した生存曲線(処理部本体)



異質性を考慮した生存曲線(HDD)

用途別の期待生存確率