Theory of Firm

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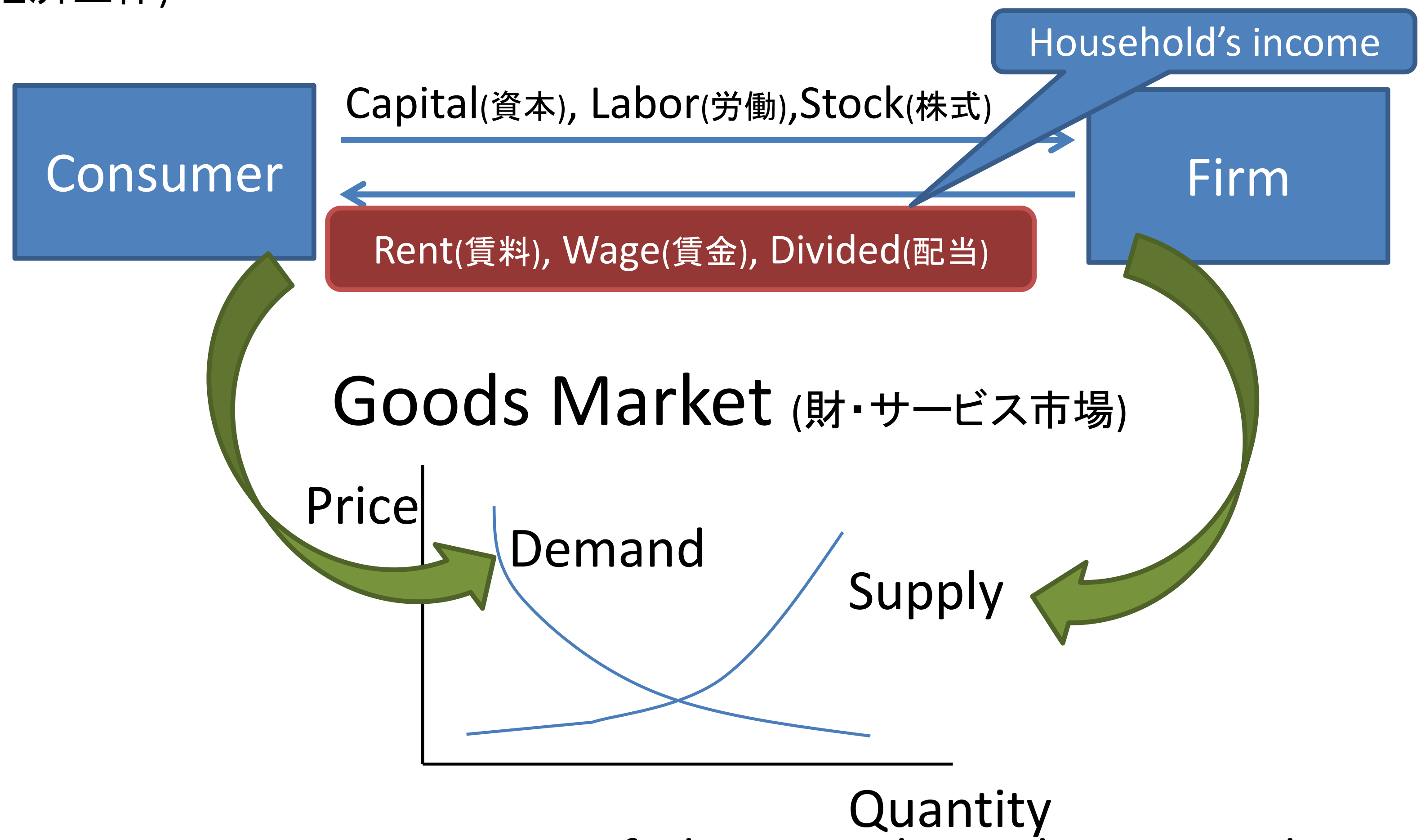
- Feature of Firm's behaviour
- 企業の行動の特徴
- Cost minimization and Profit maximization
- 費用最小化と利潤最大化
- Cost function and Profit function
- 費用関数と利潤関数
- Market Supply Curve
- 市場供給関数
- Long run Equilibrium
- 長期均衡

Feature of Consumer Behaviour

Economic Entity
(経済主体)

Firm(企業), Consumer (家計), Government



Firm = Aim to maximising profit but not always be price taker

Profit

- Profit = Revenue – Cost
  - Revenue = Sales price X Quantity

    p $\mathbf{y}$

  - Cost = Σ(Factor price X Quantity)

    w_i x_i

Constraints on Firm's behaviour

- Technological Constraints (技術的制約)
- Market Constraints (市場の制約)
  - Price mechanism that firm faces on

Market for outputs (産出物の市場)

Multiple player → Price taker

Single supplier → Monopoly (独占)

Market for the factors inputs (生産要素市場)

Multiple recipient → Price taker

Single recipient → Monopoly (独占)

→ Competitive Market

Description of Technology (1)

- Technology is system that transform factors of production (生産要素) into productions (生産物)

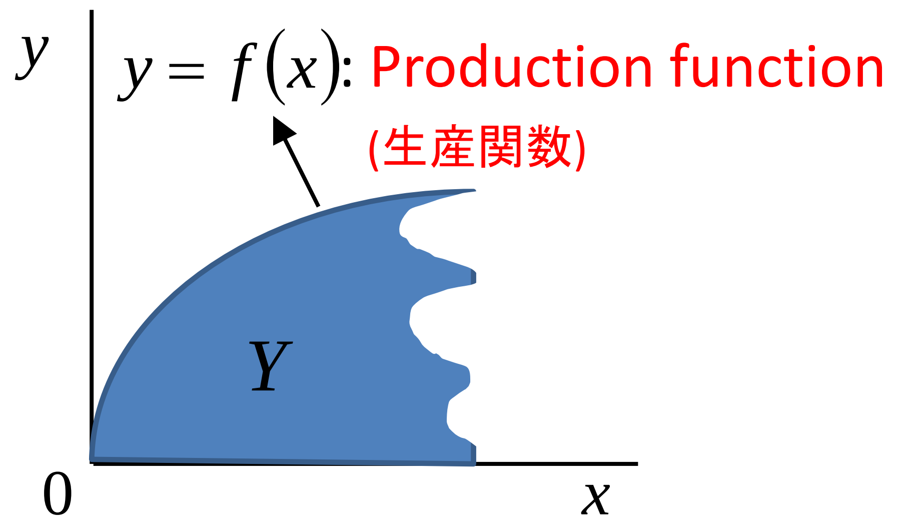
Production Possibilities Set (生産可能集合)

$$Y = \{(\mathbf{x}, \mathbf{y}), \mathbf{x} \in R_+^n, \mathbf{y} \in R_+^n\}$$

where

$\mathbf{x}$: Amount of factor inputs (input)

$\mathbf{y}$: Amount of productions (output)

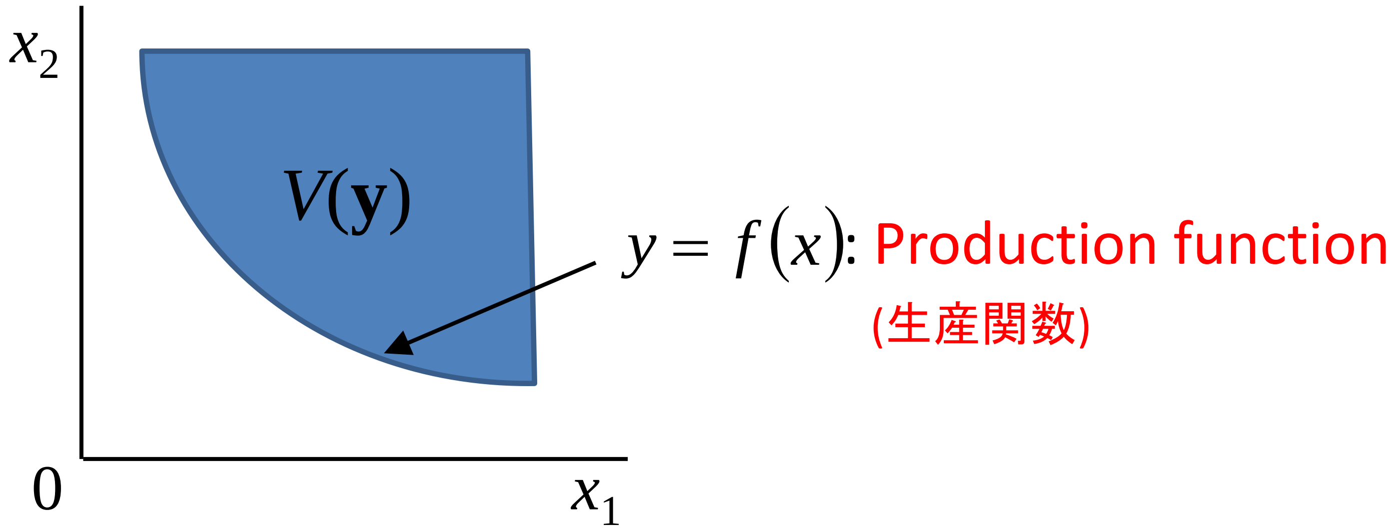


Description of Technology (2)

Input Requirement set (必要投入量集合)

$$V(\mathbf{y}) = \left\{ \mathbf{x} \in R_{+}^{n} \middle| (\mathbf{x}, \mathbf{y}) \in Y \right\}$$

> *Input requirement set* is defined as that set of inputs required to produce at least a given level of outputs, *y*



Example of Production Function

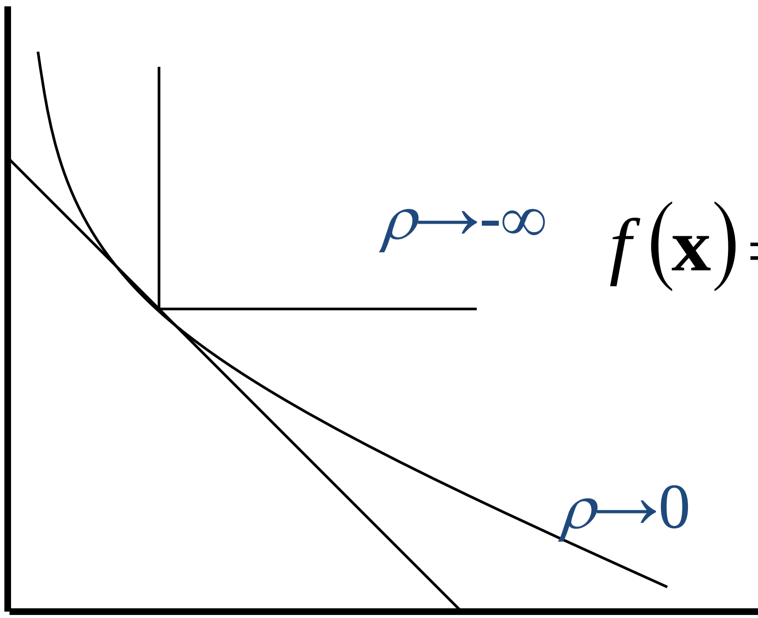
- Leontief type

- Cobb-Douglas type

- Linear type

Example: CES type production function

$$f(\mathbf{x}) = \left(a_0 + a_1 x_1^{\rho} + \cdots a_n x_n^{\rho}\right)^{1/\rho}$$



$\rho \to -\infty$ $$f(\mathbf{x}) = \min\{a_1 x_1, \ldots, a_n x_n\}$$

$\rho \to 0$ $$f(\mathbf{x}) = a_0 x_1^{a_1} \cdots x_n^{a_n}$$

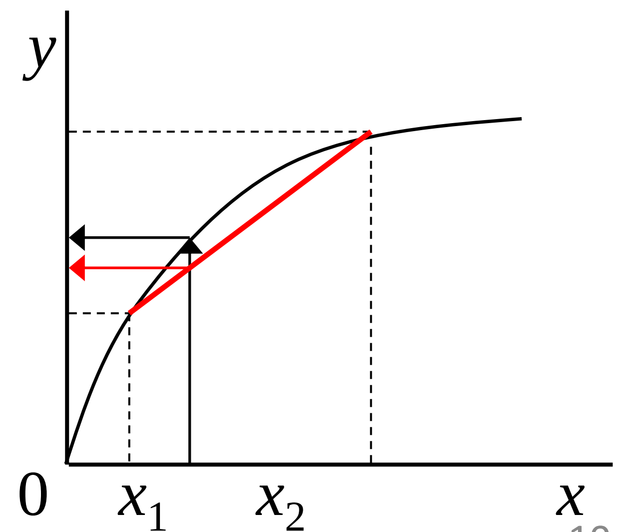
$\rho = 1$ $$f(\mathbf{x}) = a_0 + a_1 x_1 + \cdots a_n x_n$$

Feature of Production Function (Assumption)

1. $f(0) = 0$
2. $f(\mathbf{x})$ is not monotonically decreasing with regard to $\mathbf{x}$
3. $f(\mathbf{x})$ is quasiconcave function (準凹関数)

   $\Leftrightarrow V(y)$ is a convex set where

$$V(y) = \left\{ \mathbf{x} \in R_{+}^{n} \middle| y \le f(\mathbf{x}) \right\}$$



Returns to Scale

$$\forall \mathbf{x}, \mathbf{x}' \in R_+^n \;, 0 \le t \le 1$$

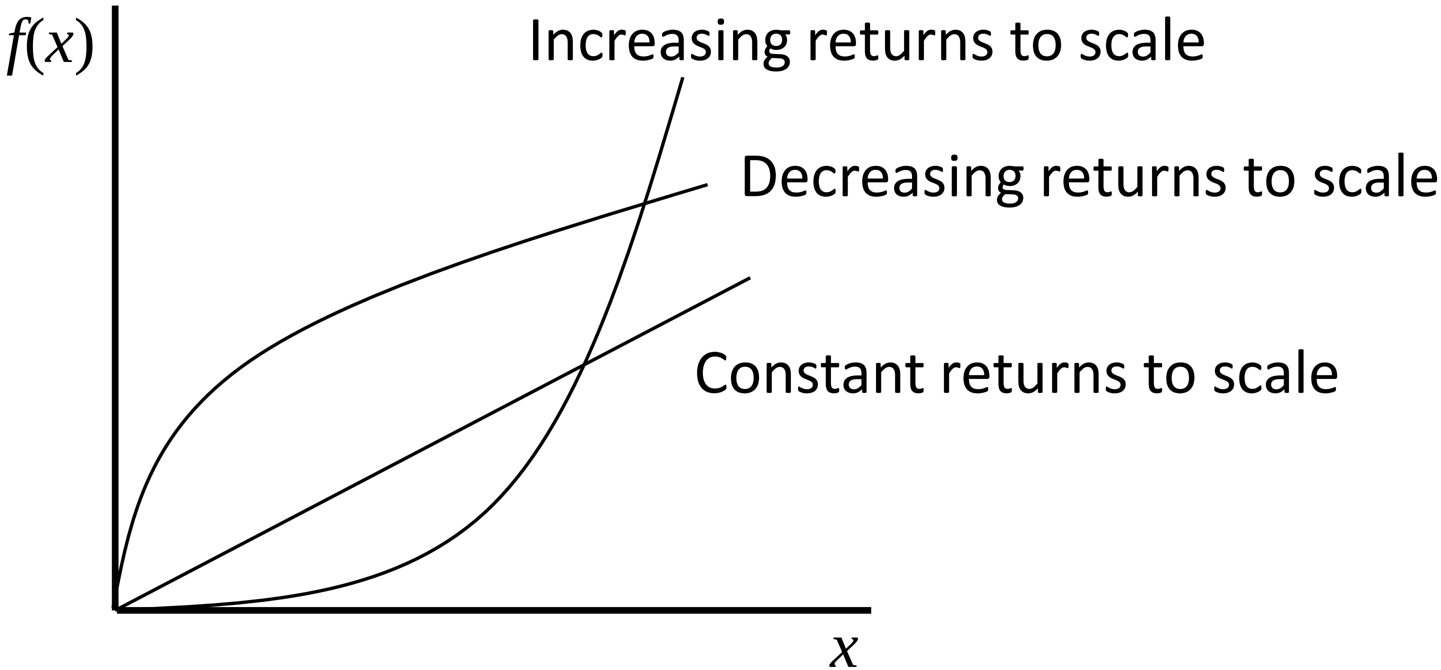
$$f\left(t\mathbf{x} + (1-t)\mathbf{x}'\right) \begin{cases} \le \\ = \\ \ge \end{cases} tf\left(\mathbf{x}\right) + \left(1-t\right) f\left(\mathbf{x}'\right)$$

– Increasing returns to scale (規模に関して収穫逓増)

– Constant returns to scale (規模に関して収穫不変)

– Decreasing returns to scale (規模に関して収穫逓増)

Example of Returns to Scale



Firm’s behaviour

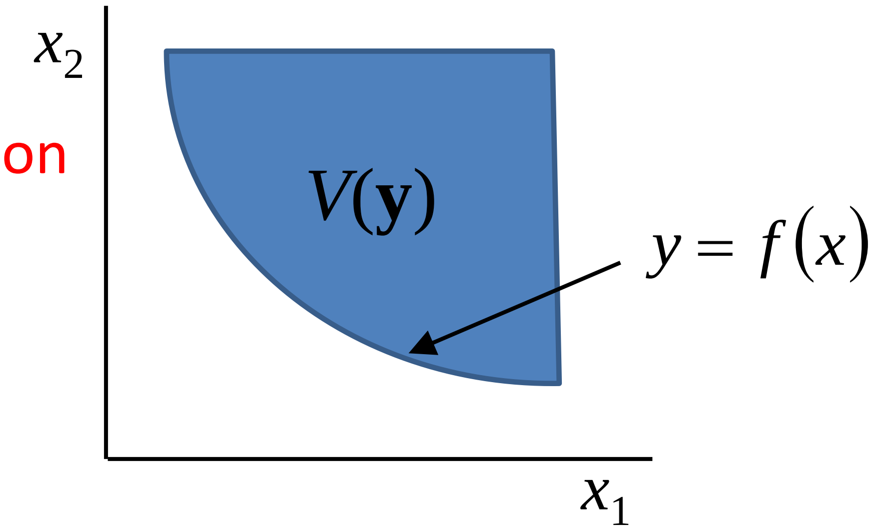
- Considering competitive firm(競争的企業)
- Profit maximisation

$$\max_{\mathbf{y},\mathbf{x}} \; p\mathbf{y} - \sum_{i=1}^{n} w_i x_i$$

Revenue　Cost

Such that

$$\mathbf{y} = f(\mathbf{x})$$: Production function
(生産関数)



Firm's behaviour (Cont.)

$$\pi(p,w) = \max_{\mathbf{x}} \left[pf(x) - \sum_{i=1}^{n} w_i x_i \right]$$

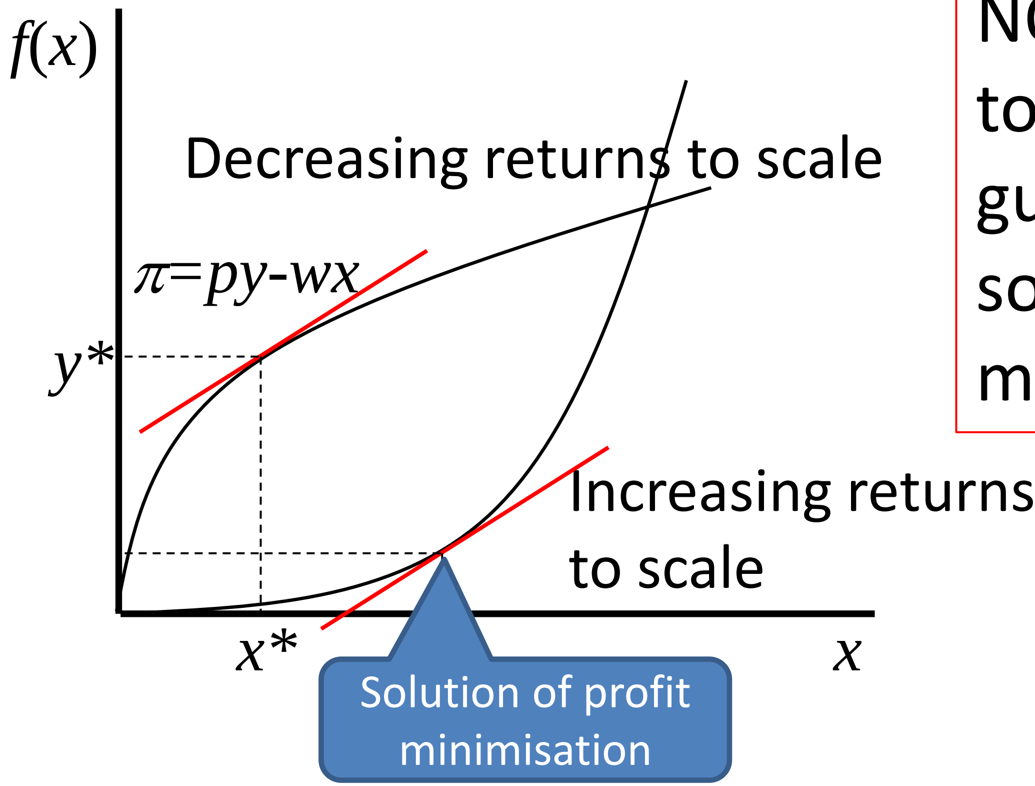
First order condition

$$p \frac{\partial f(\mathbf{x})}{\partial x_i} = w_i$$

Value of Marginal production = Factor price

限界生成物の価値　　要素価格

Returns to Scale and Profit Maximisation



Production function should NOT has increasing returns to scale in order to guarantee the existing of solution of the profit maximisation problem

* If production function has increasing returns to scale, the solution is $x = \infty$

Cost Minimisation

$$c(w,y) = \min \sum_{i=1}^{n} w_i x_i$$

Subject to $\mathbf{y} = f(\mathbf{x})$

First order condition

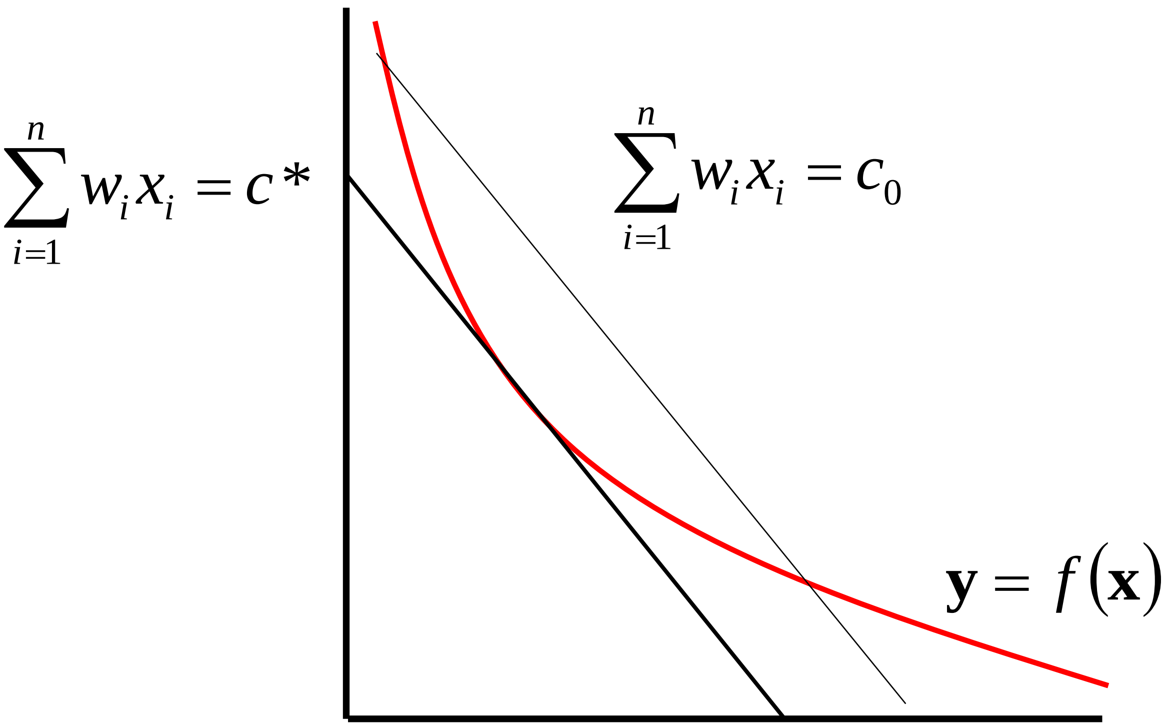
$$\begin{cases} w_i = \lambda \dfrac{\partial f(\mathbf{x})}{\partial x_i} \\ \mathbf{y} = f(\mathbf{x}) \end{cases} \Rightarrow w_i / w_j = \frac{\partial f(\mathbf{x})}{\partial x_i} \Big/ \frac{\partial f(\mathbf{x})}{\partial x_j}$$

Factor price ratio = Technical marginal rate of substitution

要素価格比

技術的限界代替率

Illustration



Conditional Factor Demand Functions
(条件付要素需要関数)

Solution of Cost Minimisation Problem;

$$x_i = x_i(\mathbf{w}, \mathbf{y})$$

Shephard's Lemma (シェパードのレンマ)

$$x_i(w, y) = \frac{\partial c(\mathbf{w}, \mathbf{y})}{\partial w_i}$$

Proof

Profit Maximisation

$$\pi(p, w) = \max\left[py - c(w, y)\right]$$

⇩

First Order Condition

$$p = \frac{\partial c(w, y)}{\partial y}$$

Factor Demand Function (要素需要関数)

Supply Function (供給関数)

Factor Demand Function $x_i = x_i(p, w)$

Supply Function $y = y(p, w)$

↓

Hotelling's Lemma (ホテリングのレンマ)

$$y(p, w) = \frac{\partial \pi(p, w)}{\partial p}$$

$$x_i(p, w) = -\frac{\partial \pi(p, w)}{\partial w_i}$$

Proof

Proof (Cont.)

Short/Long-run Cost Function
(長期・短期の費用関数)

$$c(y) = c_v(y) + F$$

Cost (費用) = Variable Cost (可変費用) + Fixed Cost (固定費用)

- Considering fixed factors of production
  → Short-run
- Not considering fixed factors of production
  → Long-run

Average Cost, Marginal Cost
(平均費用, 限界費用)

* We only consider short run cost

- Short-run Average Cost (AC) 短期平均費用

$$AC(y) = c(y)/y = c_v(y)/y + F/y$$

Short-run Average Variable Cost; AVC (短期平均可変費用)

Short-run Average Fixed Cost; AFC (短期平均固定費用)

- Short-run Marginal Cost (MC) 短期限界費用

$$MC(y) = \partial c(y)/\partial y = \partial c_v(y)/\partial y \quad (\because \partial F/\partial y = 0)$$

Average Cost, Marginal Cost (Cont)
(平均費用, 限界費用)

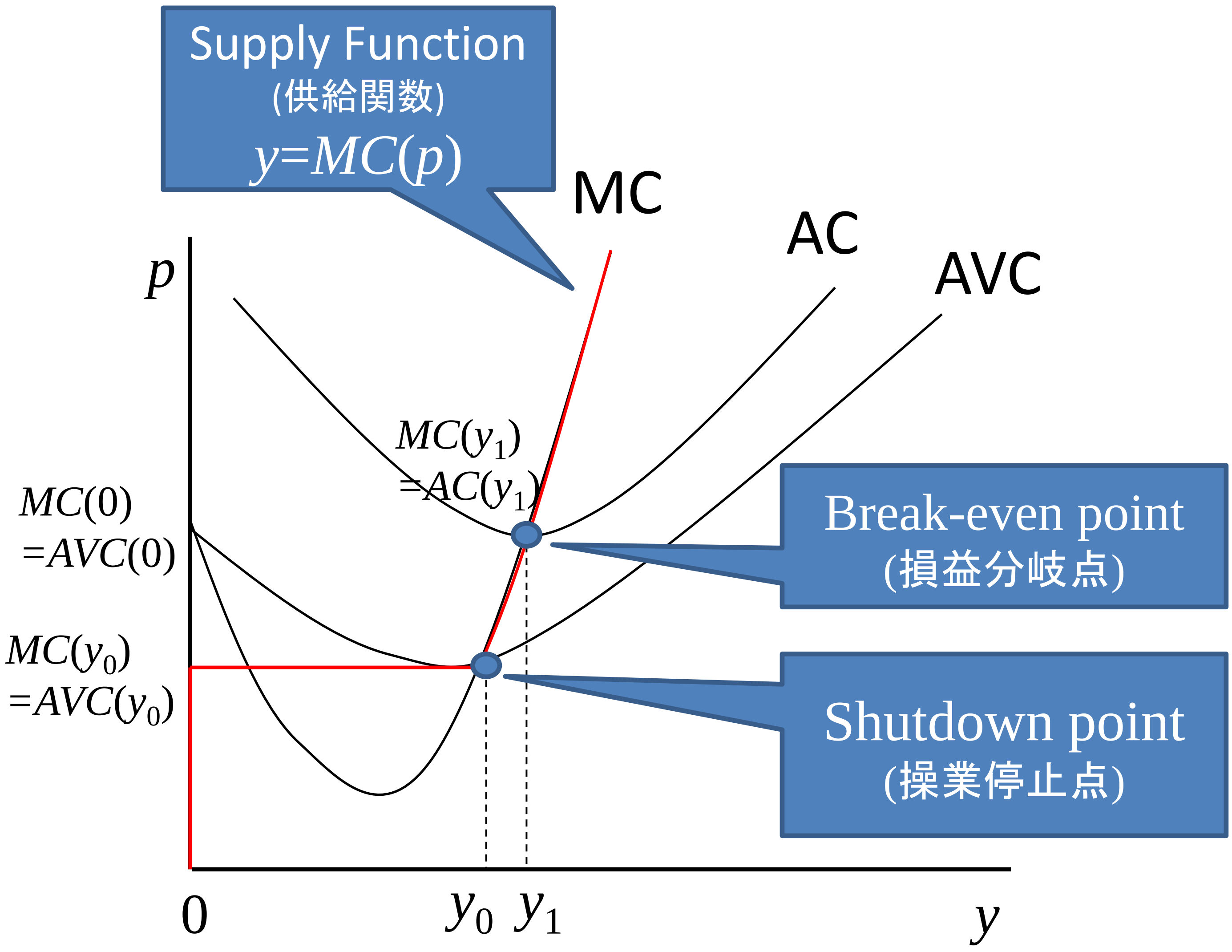
Marginal Cost curve satisfies following properties;

1. $MC(0) = AVC(0)$
2. MC curve must intersect with the AVC curve at its minimum point
3. MC curve must intersect with the AC curve at its minimum point

* Proof of property 3;

$$\min_{y} AC(y) = \min_{y} \frac{c(y)}{y} \Rightarrow \frac{\partial c(y)/y - c(y)}{y^2} = 0 \Leftrightarrow \frac{c(y)}{y} = \frac{\partial c(y)}{\partial y}$$

Illustration



Break-even point, Shutdown point and Supply Function

- Break-even point (損益分岐点)
  - Combination of price (p) and the amount of productions (y) under the zero profit

$$py - c(y) = 0 \Rightarrow p = c(y)/y = AC(y)$$

- Shutdown point (操業停止点)
  - Combination of price (p) and the amount of productions (y) where the firms have a difficulty to continue the operations

$$py - (c_v(y) + F) \geq 0 - (c_v(0) - F) \Rightarrow p \geq c_v(y)/y = AVC(y)$$

- Supply Function (供給曲線)
  - Solution of Max $py\text{-}c(y)$ ⇒

$$\begin{cases} y = MC^{-1}(p) & (p \geq \min AVC(y)) \\ y = 0 & (p < \min AVC(y)) \end{cases}$$